Caltech

A Control Barrier Perspective on Episodic Learning via Projection-to-State Safety

Andrew J. Taylor Andrew Singletary Yisong Yue Aaron D. Ames

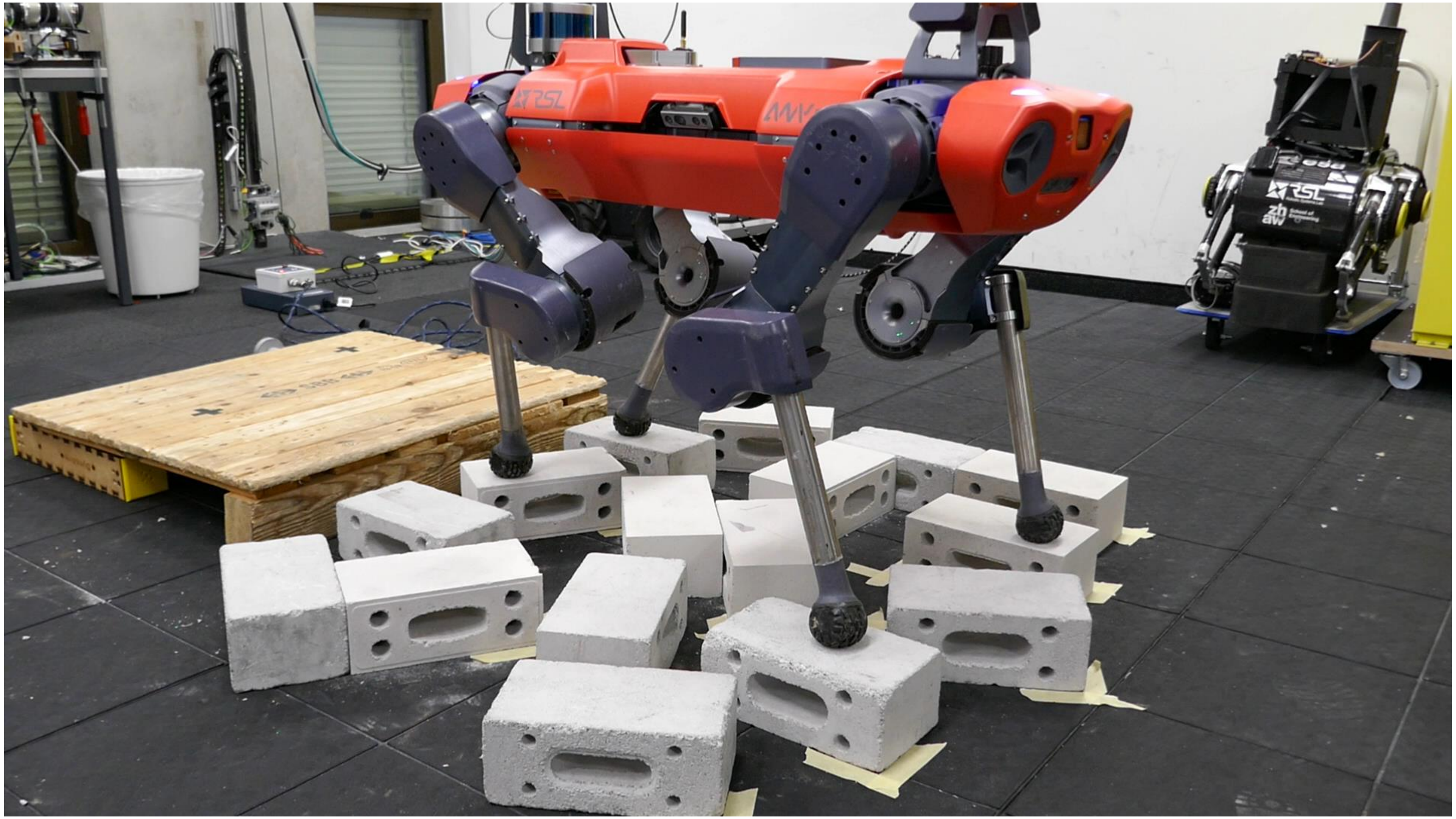
Computing and Mathematical Sciences
California Institute of Technology

December 15th, 2020

Control & Decision Conference (CDC) 2020

Control in the real world is hard

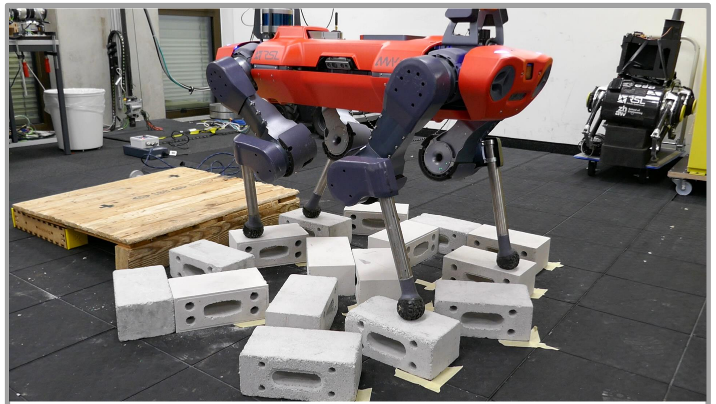
But: Pretty when it works…



[1] R. Grandia, **A. J. Taylor**, M. Hutter, A. D. Ames, Multi-Layered Safety for Legged Robotics via Control Barrier Functions and Model Predictive Control, 2020.

Claim: Need to Bridge the Gap

Caltech



$$\mathbf{k}(\mathbf{x}) = \underset{\mathbf{u} \in \mathbb{R}^m}{\operatorname{argmin}} \|\mathbf{u}\|_2^2$$
$$\text{s.t.} \quad \dot{h}(\mathbf{x}, \mathbf{u}) \geq -\alpha(h(\mathbf{x}))$$

Bridge the Gap

$$\dot{h}(\mathbf{x}, \mathbf{u}) \geq -\alpha(h(\mathbf{x})) - \delta$$

Theorems & Proofs

Experimental Realization

Contributions

Caltech

- Framework for studying impact of disturbances in a projected environment via **Projection-to-State Safety (PSSf)**
- Apply PSSf to study how error in machine learning models estimating dynamics leads to degradation in safety guarantees
- Demonstration of PSSf guarantees on safety in simulation and experimentally
- Complement existing work utilizing stability

[2] **A. J. Taylor**, V. D. Dorobantu, et. al, Episodic Learning with Control Lyapunov Functions for Uncertain Robotic Systems, 2019.
[3] **A. J. Taylor**, V. D. Dorobantu, et. al, A Control Lyapunov Perspective on Episodic Learning via Projection to State Stability, 2019.
[4] **A. J. Taylor**, et. al, Learning for Safety-Critical Control with Control Barrir Functions, 2020.

System Dynamics

Caltech

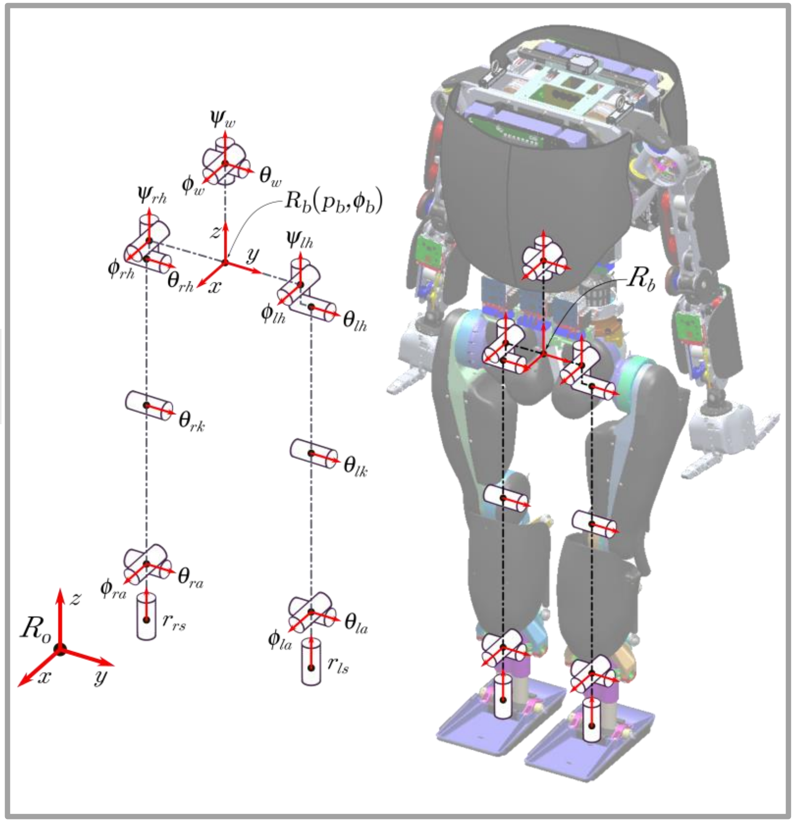
Equations of Motion

$$\dot{\widehat{\mathbf{x}}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}$$

$$\mathbf{x} \in \mathbb{R}^n \qquad \mathbf{u} \in \mathbb{R}^m$$

$$\widehat{\mathbf{f}} : \mathbb{R}^n \to \mathbb{R}^n \qquad \widehat{\mathbf{g}} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

Mathematical Model



System Model

System Dynamics

Caltech

Equations of Motion

$$\dot{\widehat{\mathbf{x}}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}$$

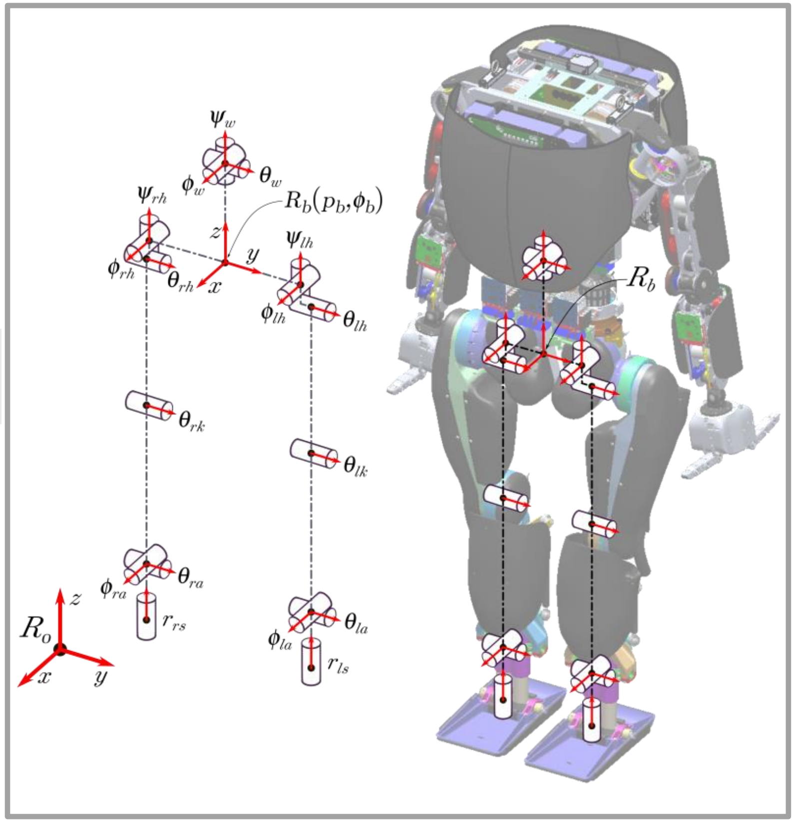
$$\mathbf{x} \in \mathbb{R}^n \qquad \mathbf{u} \in \mathbb{R}^m$$

$$\widehat{\mathbf{f}} : \mathbb{R}^n \to \mathbb{R}^n \qquad \widehat{\mathbf{g}} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

Assumptions

$\widehat{\mathbf{f}}, \widehat{\mathbf{g}}$ locally Lipschitz continuous

Mathematical Model



System Model

Control Barrier Functions (CBFs)

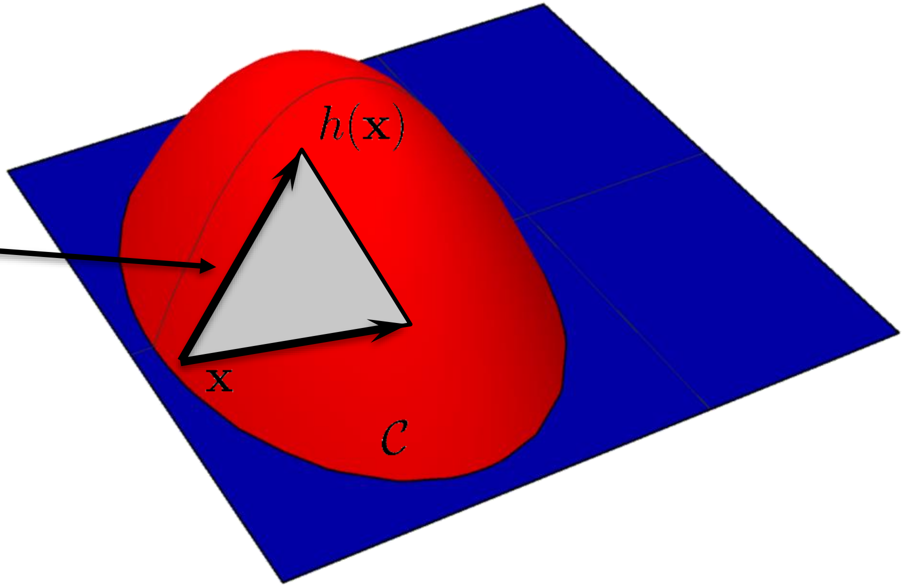
Caltech

Control Barrier Function

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n \mid h(\mathbf{x}) \geq 0\}$$

$$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}) > -\alpha(h(\mathbf{x}))$$

$$\dot{h}(\mathbf{x}, \mathbf{u}) = \frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u})$$



Control Barrier Functions (CBFs)

Caltech

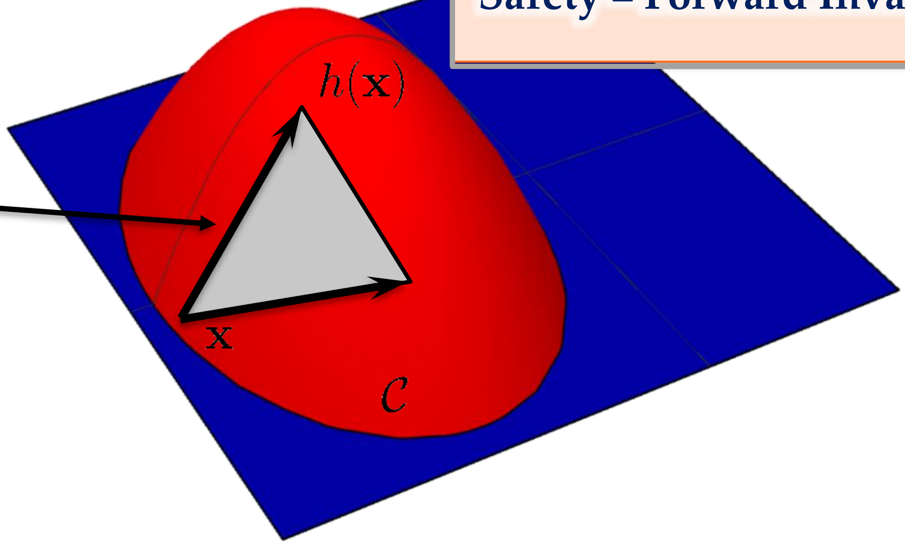
Control Barrier Function

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n \mid h(\mathbf{x}) \geq 0\}$$

$$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}) > -\alpha(h(\mathbf{x}))$$

$$\dot{h}(\mathbf{x}, \mathbf{u}) = \frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u})$$

Safety = Forward Invariance



Control Barrier Functions (CBFs)

Caltech

Control Barrier Function

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n \mid h(\mathbf{x}) \geq 0\}$$

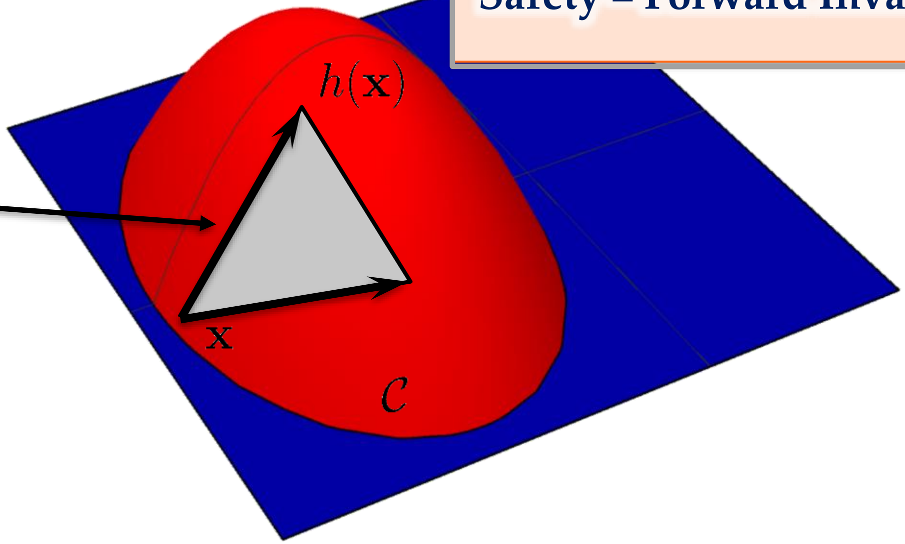
$$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}) > -\alpha(h(\mathbf{x}))$$

$$\dot{h}(\mathbf{x}, \mathbf{u}) = \frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u})$$

Feedback Controllers

[5] A. Ames, et al. Control barrier function based quadratic programs with application to adaptive cruise control, 2014.
[6] A. Ames, et al. Control barrier function based quadratic programs for safety critical systems, 2017.

Safety = Forward Invariance



Control Barrier Functions (CBFs)

Caltech

Control Barrier Function

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n \mid h(\mathbf{x}) \geq 0\}$$

$$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}) > -\alpha(h(\mathbf{x}))$$

$$\dot{h}(\mathbf{x}, \mathbf{u}) = \frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u})$$

Feedback Controllers

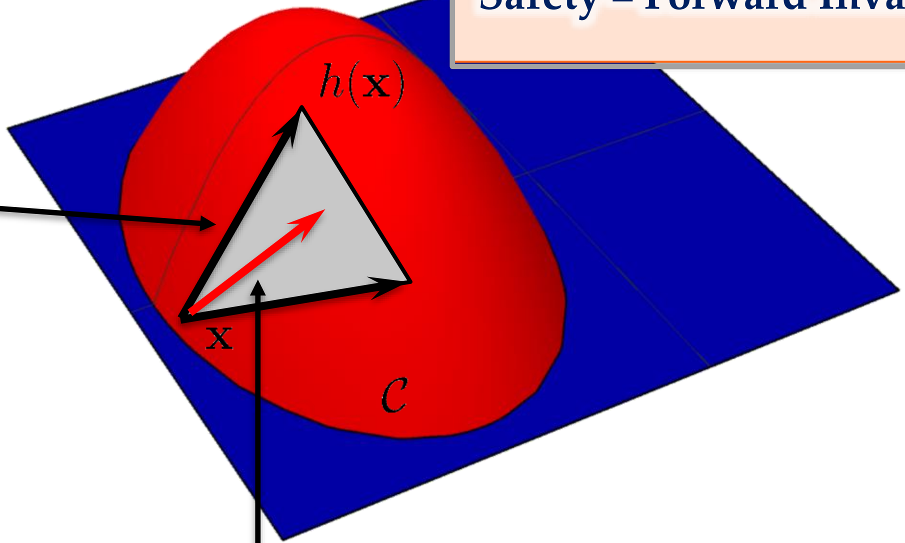
[5] A. Ames, et al. Control barrier function based quadratic programs with application to adaptive cruise control, 2014.
[6] A. Ames, et al. Control barrier function based quadratic programs for safety critical systems, 2017.

CBF Quadratic Program[6]

$$\mathbf{k}(\mathbf{x}) = \underset{\mathbf{u} \in \mathbb{R}^m}{\operatorname{argmin}} \|\mathbf{u}\|_2^2$$

$$\text{s.t.} \quad \dot{h}(\mathbf{x}, \mathbf{u}) \geq -\alpha(h(\mathbf{x}))$$

Safety = Forward Invariance



Uncertain System Dynamics

Caltech

Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\mathbf{x} \in \mathbb{R}^n \quad \mathbf{u} \in \mathbb{R}^m$$

$$\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n \quad \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

True Dynamics

Physical Robot

Uncertain System Dynamics

Caltech

Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\mathbf{x} \in \mathbb{R}^n \quad \mathbf{u} \in \mathbb{R}^m$$

$$\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n \quad \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

Methods

- Adaptive Control [7]
- System Identification [8]
- Machine Learning [9]
- High-gain control [10]

True Dynamics

Physical Robot

[7] M. Krstic, et al., Nonlinear Adaptive Control Design
[8] L. Ljung, System Identification
[9] J. Kober, et al., Reinforcement learning in robotics: A survey
[10] A. Ilchmann, et al., High-gain control without identification: a survey

Uncertain System Dynamics

Caltech

Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\mathbf{x} \in \mathbb{R}^n \quad \mathbf{u} \in \mathbb{R}^m$$

$$\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n \quad \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

Methods

- Adaptive Control
- System Identification
- Machine Learning
- High-gain control

True Dynamics

Physical Robot

Uncertain System Dynamics

Caltech

Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\mathbf{x} \in \mathbb{R}^n \quad \mathbf{u} \in \mathbb{R}^m$$

$$\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n \quad \mathbf{g} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$$

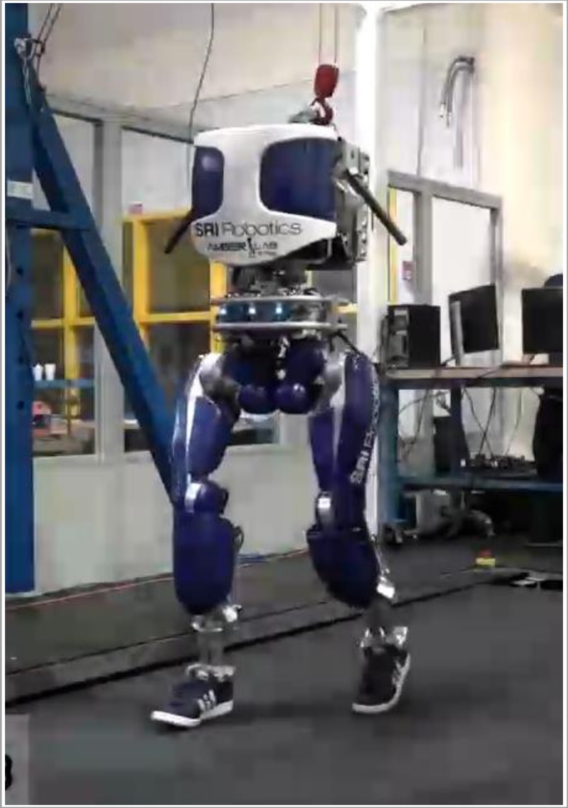
Assumptions[4]

$\mathbf{f}$, $\mathbf{g}$ locally Lipschitz continuous

$$\inf_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}) \geq -\alpha(h(\mathbf{x}))$$

$$\dot{h}(\mathbf{x}, \mathbf{u}) = \frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u})$$

True Dynamics



Physical Robot

[4] **A. J. Taylor**, A. Singletary, Y. Yue, A. D. Ames, Learning for Safety-Critical Control with Control Barrier Functions, 2020.

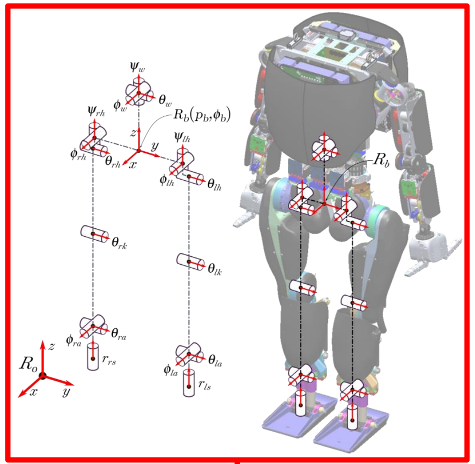
Dynamics Learning

Caltech

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

Dynamics Learning

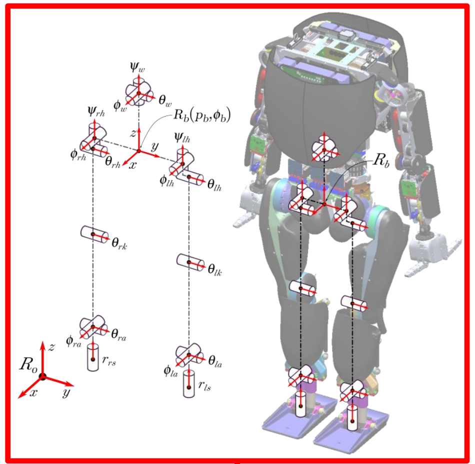
Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm \left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} \right)$$

Dynamics Learning



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

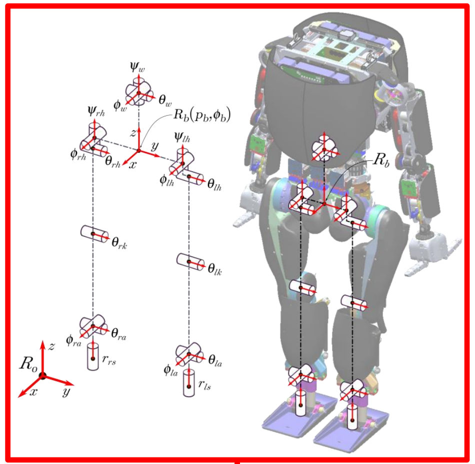
$$\pm \left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} \right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

Dynamics Learning

Learn the dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

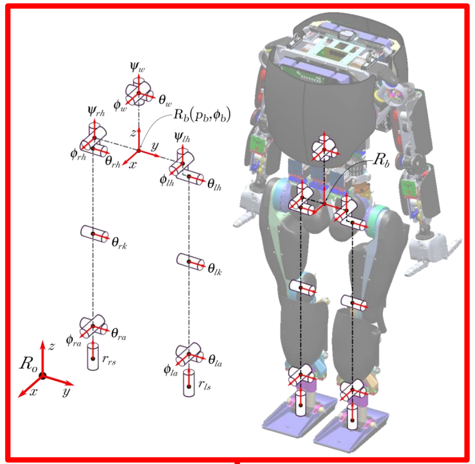
$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

Dynamics Learning

Learn the residual dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

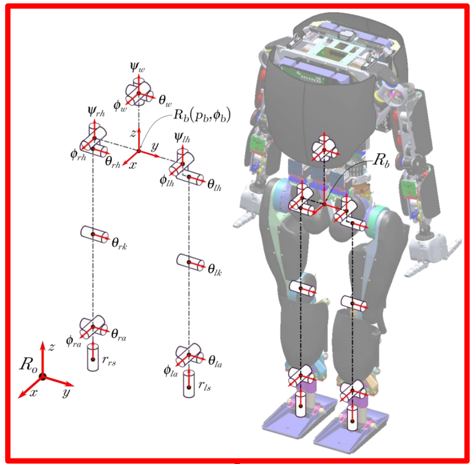
$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

$$\pm\left(\widehat{\mathbf{b}}(\mathbf{x}) + \widehat{\mathbf{A}}(\mathbf{x})\mathbf{u}\right)$$

Dynamics Learning

Learn the residual dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x}))}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

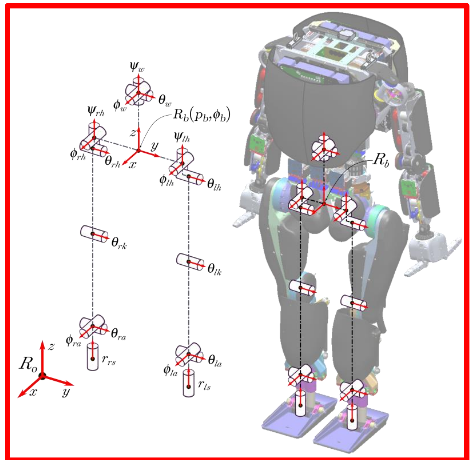
$$\pm\left(\widehat{\mathbf{b}}(\mathbf{x}) + \widehat{\mathbf{A}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{b}}(\mathbf{x}) + \left(\widehat{\mathbf{g}}(\mathbf{x}) + \widehat{\mathbf{A}}(\mathbf{x})\right)\mathbf{u} + \left(\mathbf{A}(\mathbf{x}) - \widehat{\mathbf{A}}(\mathbf{x})\right)\mathbf{u} + \mathbf{b}(\mathbf{x}) - \widehat{\mathbf{b}}(\mathbf{x})$$

Dynamics Learning

Learn the residual dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

$$\pm\left(\widehat{\mathbf{b}}(\mathbf{x}) + \widehat{\mathbf{A}}(\mathbf{x})\mathbf{u}\right)$$

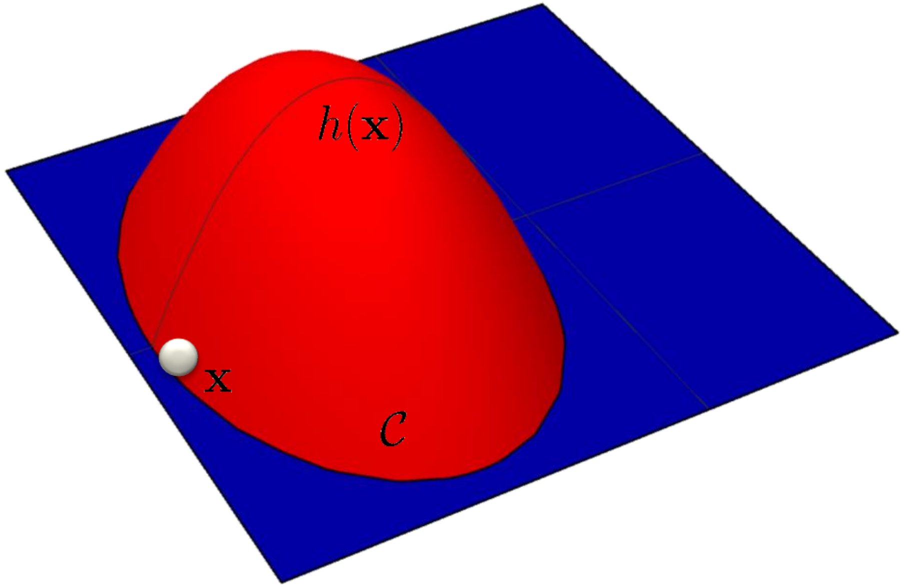
$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{b}}(\mathbf{x}) + \left(\widehat{\mathbf{g}}(\mathbf{x}) + \widehat{\mathbf{A}}(\mathbf{x})\right)\mathbf{u} + \left(\mathbf{A}(\mathbf{x}) - \widehat{\mathbf{A}}(\mathbf{x})\right)\mathbf{u} + \mathbf{b}(\mathbf{x}) - \widehat{\mathbf{b}}(\mathbf{x})$$

$$\mathbf{d} = \left(\mathbf{A}(\mathbf{x}) - \widehat{\mathbf{A}}(\mathbf{x})\right)\mathbf{u} + \mathbf{b}(\mathbf{x}) - \widehat{\mathbf{b}}(\mathbf{x})$$

Input-to-State Safety

Input-to-State Safety (ISSf)[9]

$$\mathcal{C}_{\mathbf{d}} \triangleq \{\mathbf{x} \in \mathbb{R}^n \;:\; h(\mathbf{x}) + \gamma(\|\mathbf{d}\|_\infty) \geq 0\}$$



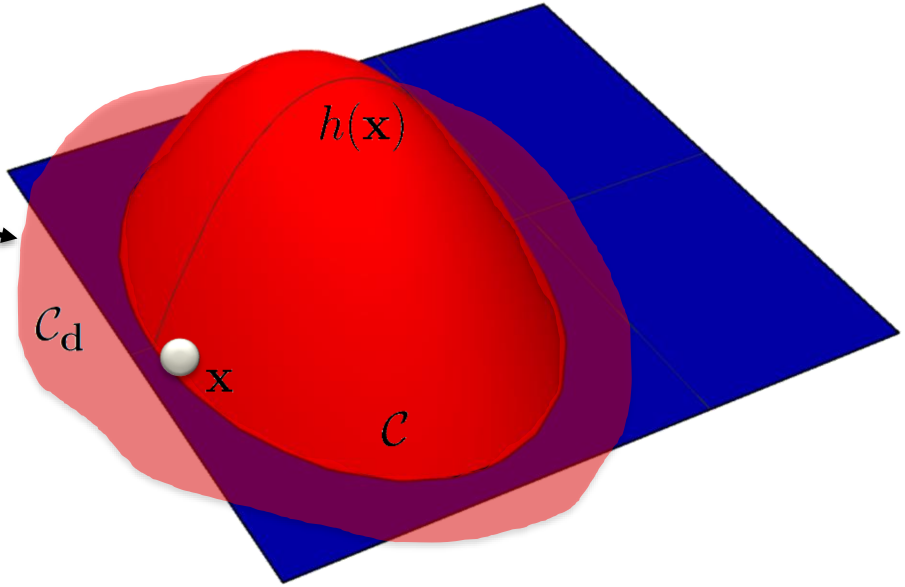
[11] S. Kolathaya, A. Ames, Input to State Safety with Control barrier functions, 2018.

Input-to-State Safety

Input-to-State Safety (ISSf)[9]

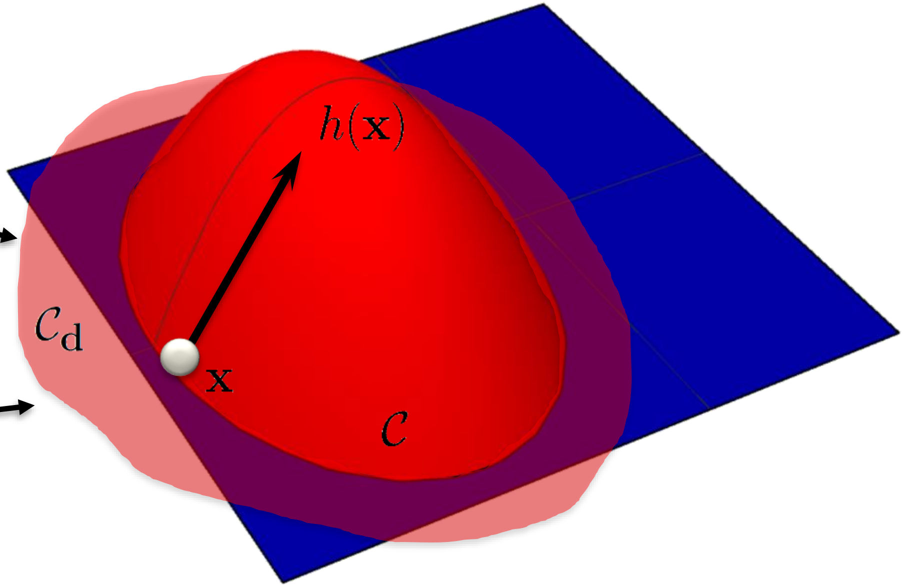
$$\mathcal{C}_\mathbf{d} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) + \gamma(\|\mathbf{d}\|_\infty) \geq 0\}$$

$$\mathcal{C} \subset \mathcal{C}_\mathbf{d}$$



[11] S. Kolathaya, A. Ames, Input to State Safety with Control barrier functions, 2018.

Input-to-State Safety (ISSf)[9]
$\mathcal{C}_\mathbf{d} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) + \gamma(\|\mathbf{d}\|_\infty) \geq 0\}$ $\mathcal{C} \subset \mathcal{C}_\mathbf{d}$
ISSf-CBF
$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}, \mathbf{d}) > -\alpha(h(\mathbf{x})) - \iota(\|\mathbf{d}\|)$



[11] S. Kolathaya, A. Ames, Input to State Safety with Control barrier functions, 2018.

Input-to-State Safety

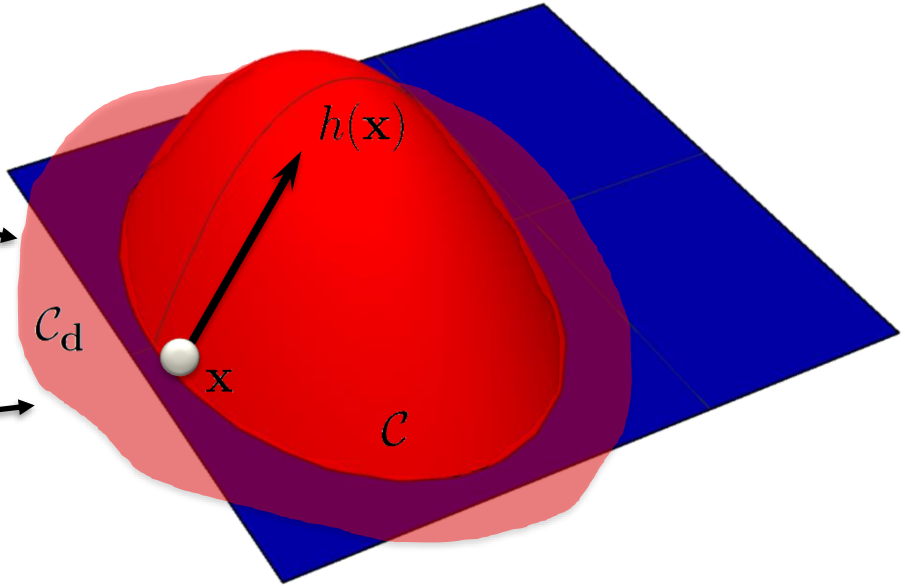
Input-to-State Safety (ISSf)[9]

$$\mathcal{C}_{\mathbf{d}} \triangleq \{\mathbf{x} \in \mathbb{R}^n \; : \; h(\mathbf{x}) + \gamma(\|\mathbf{d}\|_\infty) \geq 0\}$$

$$\mathcal{C} \subset \mathcal{C}_{\mathbf{d}}$$

ISSf-CBF

$$\sup_{\mathbf{u} \in \mathbb{R}^m} \dot{h}(\mathbf{x}, \mathbf{u}, \mathbf{d}) > -\alpha(h(\mathbf{x})) - \iota(\|\mathbf{d}\|)$$



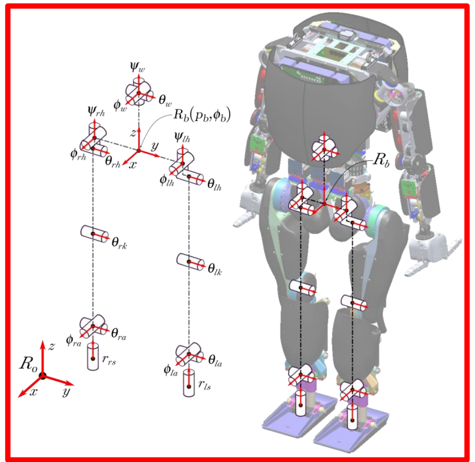
ISSf-CBF → ISSf

[11] S. Kolathaya, A. Ames, Input to State Safety with Control barrier functions, 2018.

CBF Derivative Learning

Learn the residual dynamics

Caltech



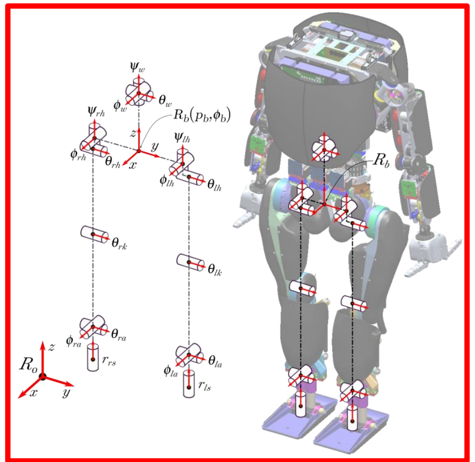
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

CBF Derivative Learning

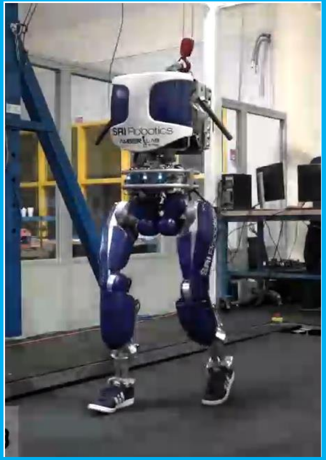
Learn the residual dynamics



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

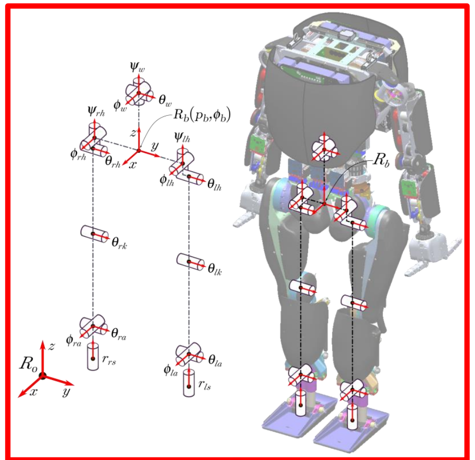


$$\dot{h}(\mathbf{x},\mathbf{u}) = \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)}_{\widehat{\dot{h}}(\mathbf{x},\mathbf{u})} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{A}(\mathbf{x})}_{\mathbf{a}(\mathbf{x})^\top}\mathbf{u} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{b}(\mathbf{x})}_{b(\mathbf{x})}$$

CBF Derivative Learning

Learn the residual CBF dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

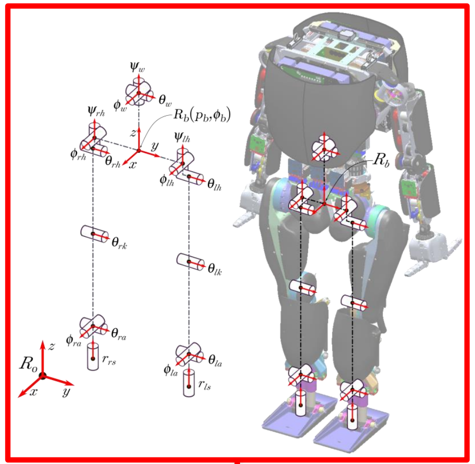
$$\dot{h}(\mathbf{x},\mathbf{u}) = \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)}_{\widehat{\dot{h}}(\mathbf{x},\mathbf{u})} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{A}(\mathbf{x})}_{\mathbf{a}(\mathbf{x})^\top}\mathbf{u} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{b}(\mathbf{x})}_{b(\mathbf{x})}$$

$$\pm\left(\widehat{b}(\mathbf{x}) + \widehat{\mathbf{a}}(\mathbf{x})^\top\mathbf{u}\right)$$

CBF Derivative Learning

Learn the residual CBF dynamics

Caltech



$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$$

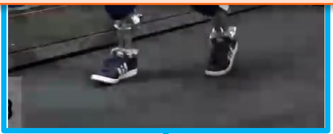
$$\pm\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)$$

$$\dot{\mathbf{x}} = \widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u} + \underbrace{\left(\mathbf{g}(\mathbf{x}) - \widehat{\mathbf{g}}(\mathbf{x})\right)}_{\mathbf{A}(\mathbf{x})}\mathbf{u} + \underbrace{\mathbf{f}(\mathbf{x}) - \widehat{\mathbf{f}}(\mathbf{x})}_{\mathbf{b}(\mathbf{x})}$$

$$\dot{h}(\mathbf{x},\mathbf{u}) = \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\left(\widehat{\mathbf{f}}(\mathbf{x}) + \widehat{\mathbf{g}}(\mathbf{x})\mathbf{u}\right)}_{\widehat{\dot{h}}(\mathbf{x},\mathbf{u})} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{A}(\mathbf{x})}_{\mathbf{a}(\mathbf{x})^\top}\mathbf{u} + \underbrace{\frac{\partial h}{\partial \mathbf{x}}(\mathbf{x})\mathbf{b}(\mathbf{x})}_{b(\mathbf{x})}$$

$$\delta = \left(\mathbf{a}(\mathbf{x}) - \widehat{\mathbf{a}}(\mathbf{x})\right)^\top \mathbf{u} + b(\mathbf{x}) - \widehat{b}(\mathbf{x})$$

How do we quantify the impact of this disturbance in a projected environment?



$$\pm\left(\widehat{b}(\mathbf{x}) + \widehat{\mathbf{a}}(\mathbf{x})^\top \mathbf{u}\right)$$

Projection-to-State Safety

Caltech

Projection
$\mathbf{\Pi} : \mathbb{R}^n \rightarrow \mathbb{R}^k$ $\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$

Projection-to-State Safety

Caltech

Projection

$$\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$$

Projected Dynamics

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta}$$

Projection-to-State Safety

Caltech

Projection
$\mathbf{\Pi}:\mathbb{R}^n\to\mathbb{R}^k$
$\mathbf{y}=\mathbf{\Pi}(\mathbf{x})$

Projected Dynamics
$\dot{\mathbf{y}}=\mathbf{f}_{\mathbf{y}}(\mathbf{x})+\mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u}+\boldsymbol{\delta}$

Definition 6 (*Projection-to-State Safety*)**.** The closed-loop system (8) is *projection-to-state safe* (PSSf) on $\mathcal{C}$ with respect to the projection $\mathbf{\Pi}$ and projected disturbances $\boldsymbol{\delta}$ if there exists $\overline{\delta}>0$ and $\gamma\in\mathcal{K}_\infty$ such that the set $\mathcal{C}_{\boldsymbol{\delta}}\supset\mathcal{C}$,

$$\mathcal{C}_{\boldsymbol{\delta}}\triangleq\{\mathbf{x}\in\mathbb{R}^n : h(\mathbf{x})+\gamma(\|\boldsymbol{\delta}\|_\infty)\geq 0\},$$
$$\partial\mathcal{C}_{\boldsymbol{\delta}}\triangleq\{\mathbf{x}\in\mathbb{R}^n : h(\mathbf{x})+\gamma(\|\boldsymbol{\delta}\|_\infty)=0\},$$
$$\mathrm{Int}(\mathcal{C}_{\boldsymbol{\delta}})\triangleq\{\mathbf{x}\in\mathbb{R}^n : h(\mathbf{x})+\gamma(\|\boldsymbol{\delta}\|_\infty)>0\},$$

Projection-to-State Safety

Caltech

Projection

$$\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$$

Projected Dynamics

$$\dot{\mathbf{y}} = \mathbf{f_y}(\mathbf{x}) + \mathbf{g_y}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta}$$

Definition 6 (*Projection-to-State Safety*)**.** The closed-loop system (8) is *projection-to-state safe* (PSSf) on $\mathcal{C}$ with respect to the projection $\mathbf{\Pi}$ and projected disturbances $\boldsymbol{\delta}$ if there exists $\overline{\delta} > 0$ and $\gamma \in \mathcal{K}_\infty$ such that the set $\mathcal{C}_{\boldsymbol{\delta}} \supset \mathcal{C}$,

$$\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) \geq 0\},$$

$$\partial\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) = 0\},$$

$$\mathrm{Int}(\mathcal{C}_{\boldsymbol{\delta}}) \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) > 0\},$$

Projection-to-State Safety

Caltech

Projection

$$\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

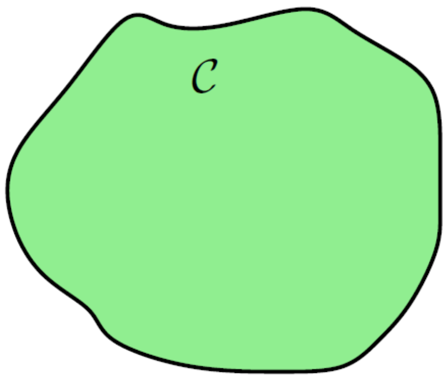
$$\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$$

Projected Dynamics

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta}$$

Definition 6 (*Projection-to-State Safety*). The closed-loop system (8) is *projection-to-state safe* (PSSf) on $\mathcal{C}$ with respect to the projection $\mathbf{\Pi}$ and projected disturbances $\boldsymbol{\delta}$ if there exists $\overline{\delta} > 0$ and $\gamma \in \mathcal{K}_\infty$ such that the set $\mathcal{C}_{\boldsymbol{\delta}} \supset \mathcal{C}$,

$$\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) \geq 0\},$$
$$\partial\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) = 0\},$$
$$\mathrm{Int}(\mathcal{C}_{\boldsymbol{\delta}}) \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) > 0\},$$



Projection-to-State Safety

Caltech

Projection

$$\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

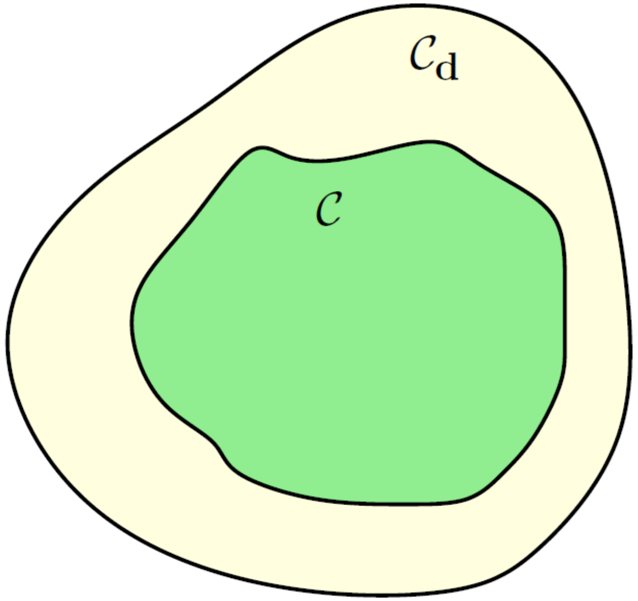
$$\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$$

Projected Dynamics

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta}$$

Definition 6 (*Projection-to-State Safety*). The closed-loop system (8) is *projection-to-state safe* (PSSf) on $\mathcal{C}$ with respect to the projection $\mathbf{\Pi}$ and projected disturbances $\boldsymbol{\delta}$ if there exists $\overline{\delta} > 0$ and $\gamma \in \mathcal{K}_\infty$ such that the set $\mathcal{C}_\delta \supset \mathcal{C}$,

$$\mathcal{C}_\delta \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) \geq 0\},$$
$$\partial\mathcal{C}_\delta \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) = 0\},$$
$$\mathrm{Int}(\mathcal{C}_\delta) \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) > 0\},$$



Projection-to-State Safety

Caltech

Projection

$\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$

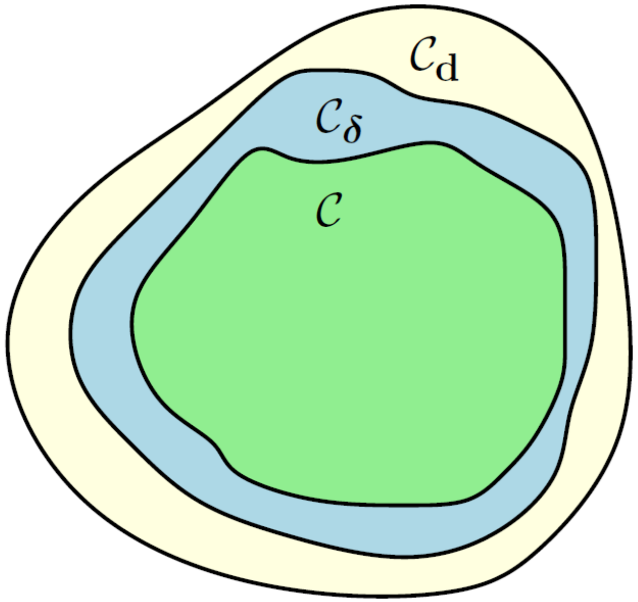
$\mathbf{y} = \mathbf{\Pi}(\mathbf{x})$

Projected Dynamics

$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta}$

Definition 6 (*Projection-to-State Safety*). The closed-loop system (8) is *projection-to-state safe* (PSSf) on $\mathcal{C}$ with respect to the projection $\mathbf{\Pi}$ and projected disturbances $\boldsymbol{\delta}$ if there exists $\bar{\delta} > 0$ and $\gamma \in \mathcal{K}_\infty$ such that the set $\mathcal{C}_{\boldsymbol{\delta}} \supset \mathcal{C}$,

$$\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) \geq 0\},$$
$$\partial\mathcal{C}_{\boldsymbol{\delta}} \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) = 0\},$$
$$\mathrm{Int}(\mathcal{C}_{\boldsymbol{\delta}}) \triangleq \{\mathbf{x} \in \mathbb{R}^n : h(\mathbf{x}) + \gamma(\|\boldsymbol{\delta}\|_\infty) > 0\},$$



Compatible Projection

Caltech

Compatible Projection

Definition 7 (*Compatible Projection*)**.** A function $h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$ is said to be a *compatible projection* for the function $h : \mathbb{R}^n \to \mathbb{R}$ with respect to the projection $\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$ if there exists $\underline{\sigma}, \overline{\sigma} \in \mathcal{K}_{\infty,e}$ such that for all $\mathbf{x} \in \mathbb{R}^n$:

$$\underline{\sigma}(h(\mathbf{x})) \leq h_{\mathbf{\Pi}}(\mathbf{\Pi}(\mathbf{x})) \leq \overline{\sigma}(h(\mathbf{x})).$$

Compatible Projection

Caltech

Compatible Projection

Definition 7 (*Compatible Projection*)**.** A function $h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$ is said to be a *compatible projection* for the function $h : \mathbb{R}^n \to \mathbb{R}$ with respect to the projection $\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$ if there exists $\underline{\sigma}, \overline{\sigma} \in \mathcal{K}_{\infty,e}$ such that for all $\mathbf{x} \in \mathbb{R}^n$:

$$\underline{\sigma}(h(\mathbf{x})) \leq h_{\mathbf{\Pi}}(\mathbf{\Pi}(\mathbf{x})) \leq \overline{\sigma}(h(\mathbf{x})).$$

[3] **A. J. Taylor**, V. D. Dorobantu, et al., A Control Lyapunov Perspective on Episodic Learning via Projection to State Stability, 2019.

Compatible Projection

Caltech

Compatible Projection

Definition 7 (*Compatible Projection*)**.** A function $h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$ is said to be a *compatible projection* for the function $h : \mathbb{R}^n \to \mathbb{R}$ with respect to the projection $\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$ if there exists $\underline{\sigma}, \overline{\sigma} \in \mathcal{K}_{\infty,e}$ such that for all $\mathbf{x} \in \mathbb{R}^n$:

$$\underline{\sigma}(h(\mathbf{x})) \le h_{\mathbf{\Pi}}(\mathbf{\Pi}(\mathbf{x})) \le \overline{\sigma}(h(\mathbf{x})).$$

Projected Safe Set

$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) \ge 0\}$$
$$\partial\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) = 0\}$$
$$\mathrm{Int}(\mathcal{C}_{\mathbf{\Pi}}) \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) > 0\}$$

[3] **A. J. Taylor**, V. D. Dorobantu, et al., A Control Lyapunov Perspective on Episodic Learning via Projection to State Stability, 2019.

Compatible Projection

Caltech

Compatible Projection

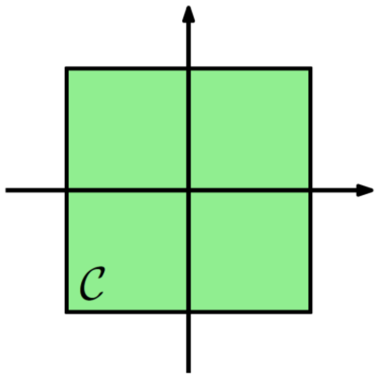
Definition 7 (*Compatible Projection*)**.** A function $h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$ is said to be a *compatible projection* for the function $h : \mathbb{R}^n \to \mathbb{R}$ with respect to the projection $\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$ if there exists $\underline{\sigma}, \overline{\sigma} \in \mathcal{K}_{\infty,e}$ such that for all $\mathbf{x} \in \mathbb{R}^n$:

$$\underline{\sigma}(h(\mathbf{x})) \leq h_{\mathbf{\Pi}}(\mathbf{\Pi}(\mathbf{x})) \leq \overline{\sigma}(h(\mathbf{x})).$$

Projected Safe Set

$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) \geq 0\}$$
$$\partial\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) = 0\}$$
$$\mathrm{Int}(\mathcal{C}_{\mathbf{\Pi}}) \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) > 0\}$$

[3] **A. J. Taylor**, V. D. Dorobantu, et al., A Control Lyapunov Perspective on Episodic Learning via Projection to State Stability, 2019.



Compatible Projection

Caltech

Compatible Projection

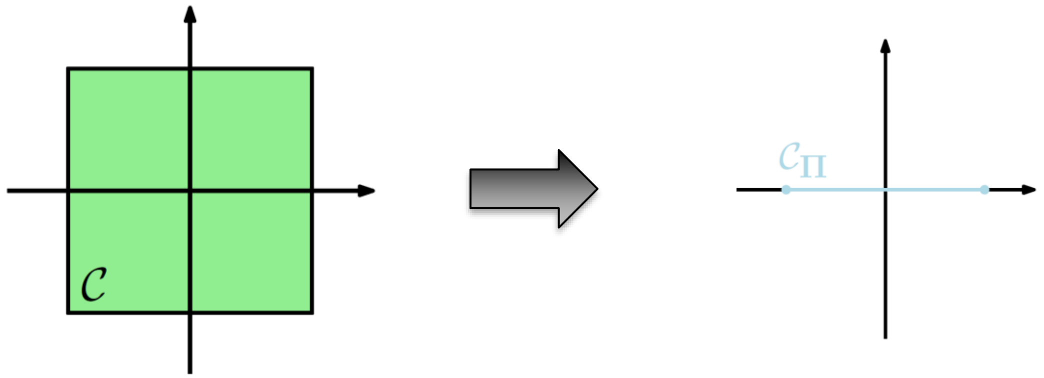
Definition 7 (*Compatible Projection*)**.** A function $h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$ is said to be a *compatible projection* for the function $h : \mathbb{R}^n \to \mathbb{R}$ with respect to the projection $\mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$ if there exists $\underline{\sigma}, \overline{\sigma} \in \mathcal{K}_{\infty,e}$ such that for all $\mathbf{x} \in \mathbb{R}^n$:

$$\underline{\sigma}(h(\mathbf{x})) \leq h_{\mathbf{\Pi}}(\mathbf{\Pi}(\mathbf{x})) \leq \overline{\sigma}(h(\mathbf{x})).$$

Projected Safe Set

$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) \geq 0\}$$
$$\partial\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) = 0\}$$
$$\mathrm{Int}(\mathcal{C}_{\mathbf{\Pi}}) \triangleq \{\mathbf{y} \in \mathbb{R}^k : h_{\mathbf{\Pi}}(\mathbf{y}) > 0\}$$

[3] **A. J. Taylor**, V. D. Dorobantu, et al., A Control Lyapunov Perspective on Episodic Learning via Projection to State Stability, 2019.



From ISSf to PSSf

Caltech

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \ (\star)$$

$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$

From ISSf to PSSf

Caltech

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \quad (\star)$$

$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$

$$\text{Projection } \mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\text{Compatible Projection } h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$$

From ISSf to PSSf

Caltech

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \quad (\star)$$

$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$

$$\text{Projection } \boldsymbol{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\text{Compatible Projection } h_{\boldsymbol{\Pi}} : \mathbb{R}^k \to \mathbb{R}$$

$$\mathcal{C}_{\boldsymbol{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k \ : \ h_{\boldsymbol{\Pi}}(\mathbf{y}) \geq 0\}$$

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta} \quad (\star\star)$$

From ISSf to PSSf

Caltech

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \quad (\star)$$
$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$
$$\text{Projection } \mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$
$$\text{Compatible Projection } h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$$
$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k \ : \ h_{\mathbf{\Pi}}(\mathbf{y}) \geq 0\}$$
$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta} \quad (\star\star)$$

⇓

$h_{\mathbf{\Pi}}$ ISSf-CBF for $(\star\star)$ on $\mathcal{C}_{\mathbf{\Pi}} \implies (\star)$ PSSf on $\mathcal{C}$

From ISSf to PSSf

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \ (\star)$$

$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$

$$\text{Projection } \mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\text{Compatible Projection } h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$$

$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k \ : \ h_{\mathbf{\Pi}}(\mathbf{y}) \geq 0\}$$

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta} \ (\star\star)$$

$h_{\mathbf{\Pi}}$ ISSf-CBF for $(\star\star)$ on $\mathcal{C}_{\mathbf{\Pi}} \implies (\star)$ PSSf on $\mathcal{C}$

CBF to PSSf

CBF h

View h as a Projection

$$y = h(\mathbf{x})$$

$$\dot{y} = f_y(\mathbf{x}) + \mathbf{g}_y(\mathbf{x})\mathbf{u} + \delta$$

From ISSf to PSSf

Caltech

Projection ISSf to PSSf

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \ (\star)$$

$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^n \ : \ h(\mathbf{x}) \geq 0\}$$

$$\text{Projection } \mathbf{\Pi} : \mathbb{R}^n \to \mathbb{R}^k$$

$$\text{Compatible Projection } h_{\mathbf{\Pi}} : \mathbb{R}^k \to \mathbb{R}$$

$$\mathcal{C}_{\mathbf{\Pi}} \triangleq \{\mathbf{y} \in \mathbb{R}^k \ : \ h_{\mathbf{\Pi}}(\mathbf{y}) \geq 0\}$$

$$\dot{\mathbf{y}} = \mathbf{f}_{\mathbf{y}}(\mathbf{x}) + \mathbf{g}_{\mathbf{y}}(\mathbf{x})\mathbf{u} + \boldsymbol{\delta} \ (\star\star)$$

$$h_{\mathbf{\Pi}} \text{ ISSf-CBF for } (\star\star) \text{ on } \mathcal{C}_{\mathbf{\Pi}} \implies (\star) \text{ PSSf on } \mathcal{C}$$

CBF to PSSf

CBF h

View h as a Projection

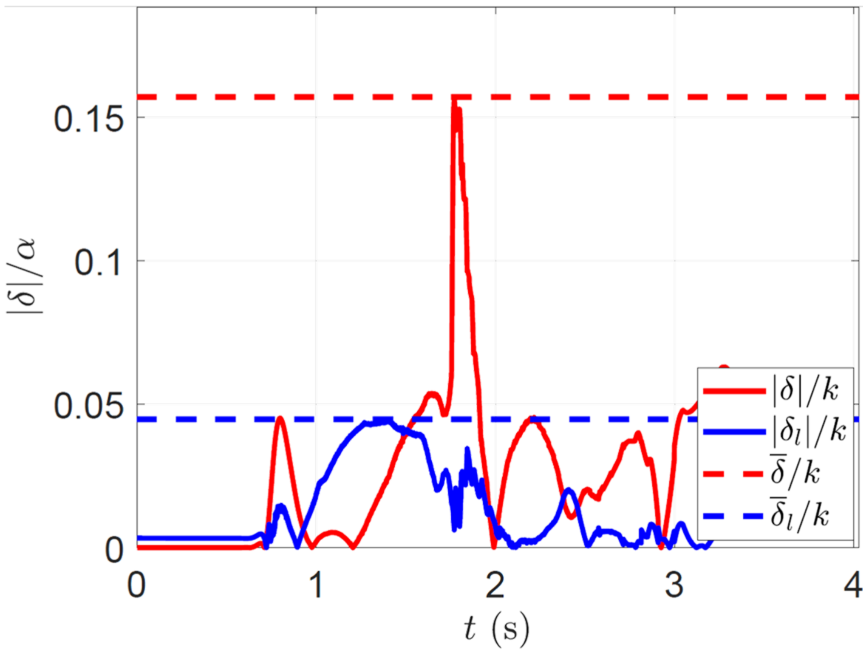
$$y = h(\mathbf{x})$$

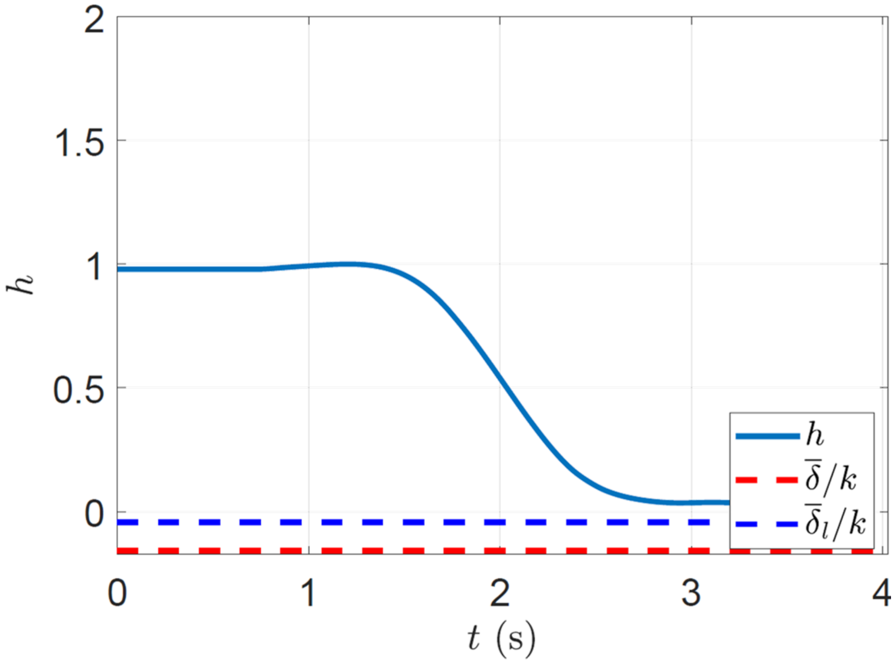
$$\dot{y} = f_y(\mathbf{x}) + \mathbf{g}_y(\mathbf{x})\mathbf{u} + \delta$$

PSSf with respect to

$$\delta = (\mathbf{a}(\mathbf{x}) - \widehat{\mathbf{a}}(\mathbf{x}))^\top \mathbf{u} + b(\mathbf{x}) - \widehat{b}(\mathbf{x})$$

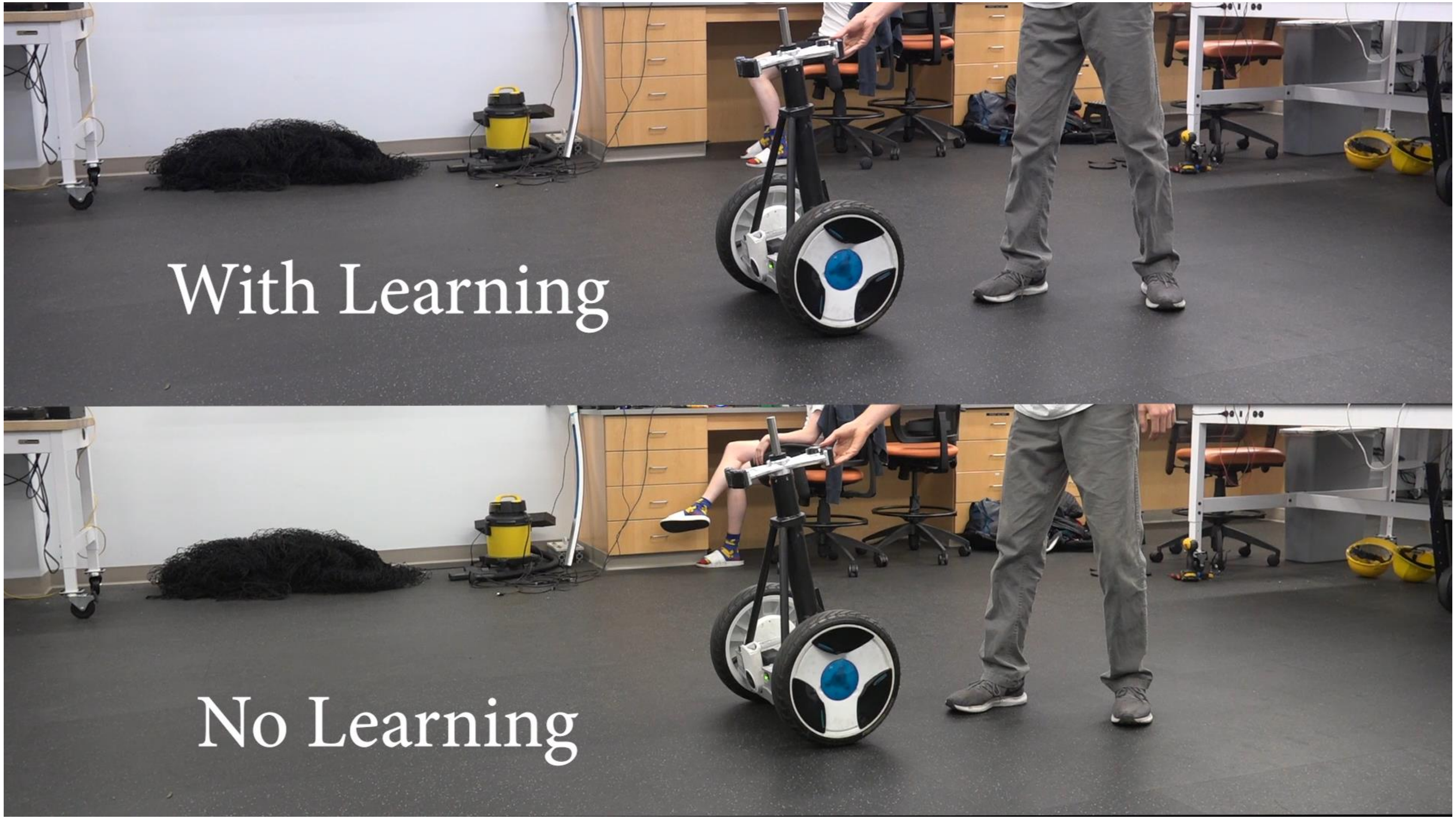
Simulation Results



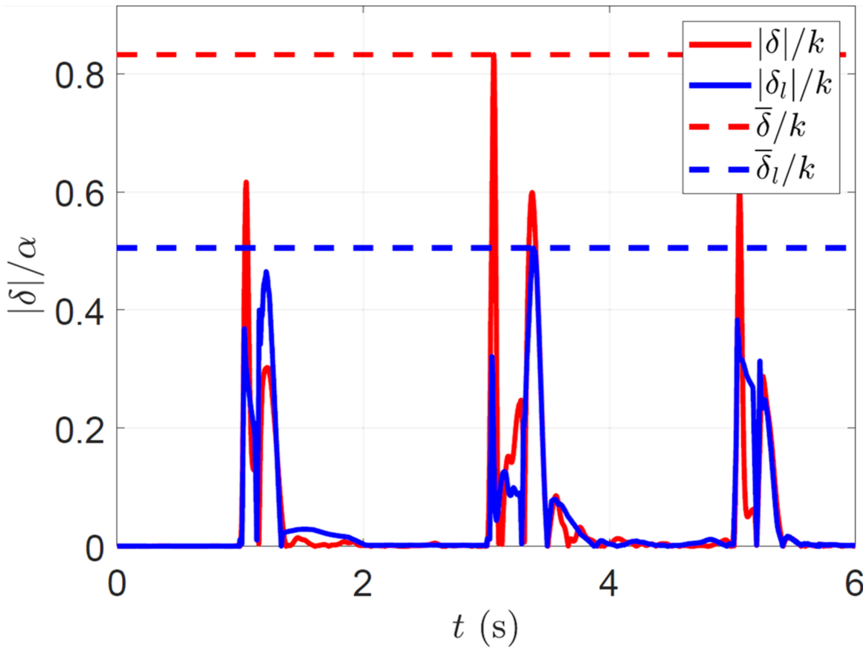


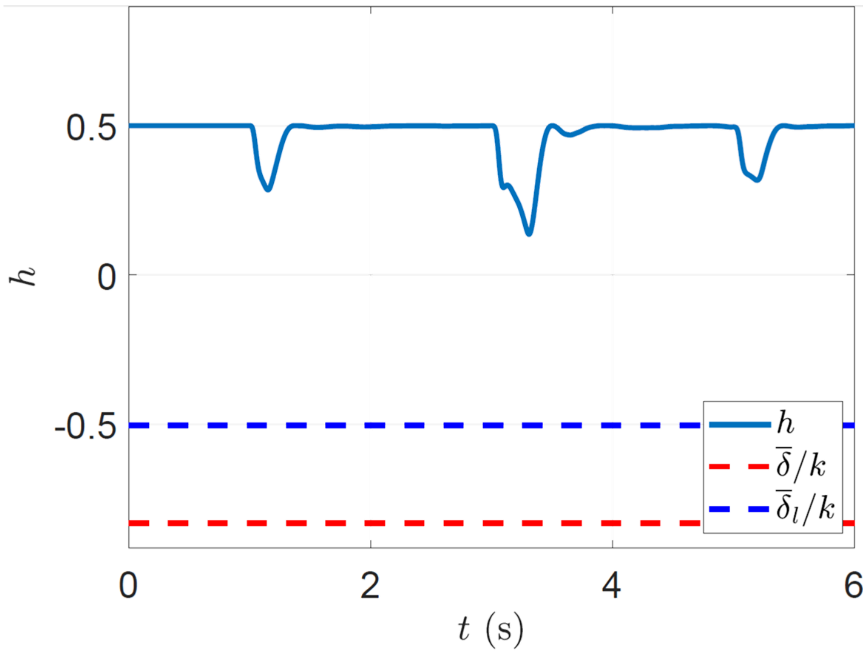
Experimental Validation

Caltech



Experimental Validation

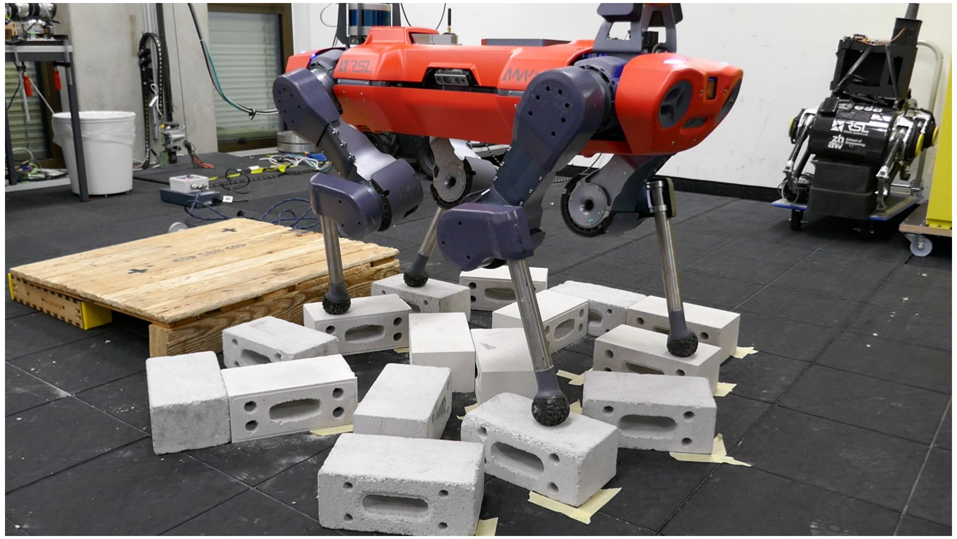




Conclusions

Caltech

- **Projection-to-State Safety** offers alternative approach for studying safety with projected disturbances
- Apply PSSf to study how machine learning error degrades safety guarantees
- PSSf behavior validated in simulation and experimentally



Thank You!

A Control Barrier Perspective on Episodic Learning via Projection to State Safety

Andrew J. Taylor   Andrew Singletary   Yisong Yue   Aaron D. Ames