ETH zürich

AMBER LAB Caltech

RSL Robotic Systems Lab

Planar Bipedal Locomotion with Nonlinear Model Predictive Control: Online Gait Generation using Whole-Body Dynamics

Manuel Yves Galliker †, N Csomay-Shanklin †, R Grandia, A J Taylor, F Farshidian, M Hutter, A D Ames

Humanoids 2022

Related Approaches – Dynamic Locomotion

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Offline Gait Synthesis using Whole-Body Dynamics

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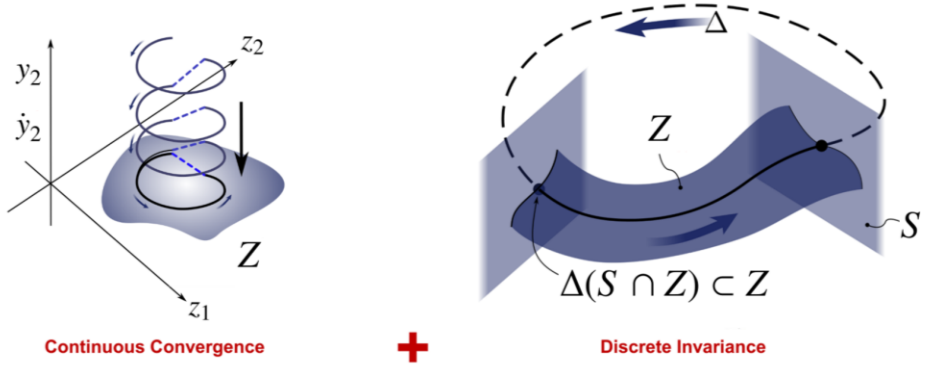
Hybrid Zero Dynamics (HZD)



Feedback Control of Dynamic Bipedal Robot Locomotion, Eric R. Westervelt, 2007

- Find periodic trajectory of actuated outputs $\boldsymbol{y_d}(\boldsymbol{q}, \boldsymbol{\alpha})$ s.t. unactuated DoF exhibit stable periodic behavior

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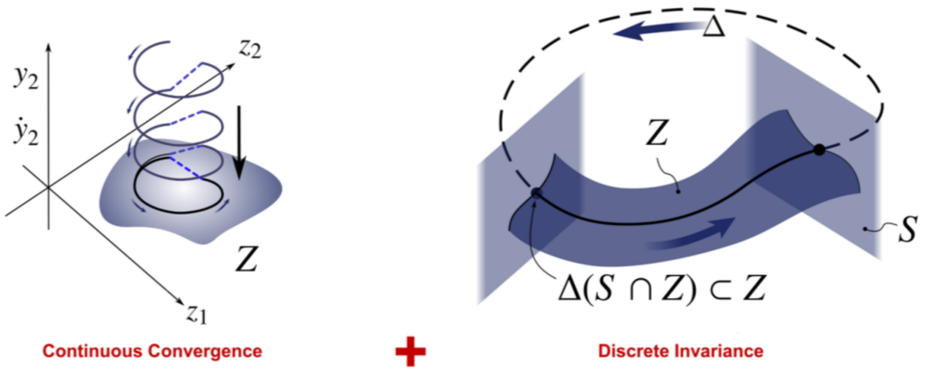

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Precomputed Stable Periodic Trajectories

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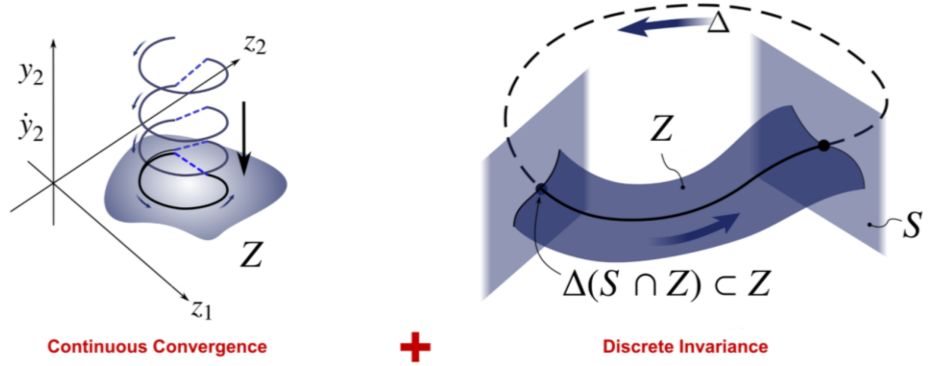

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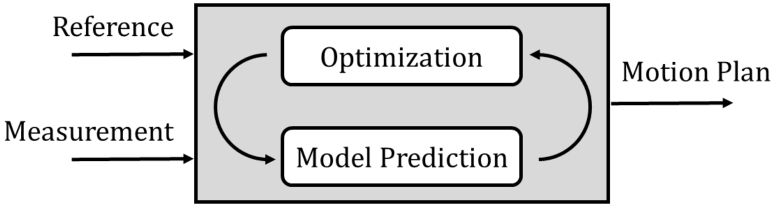
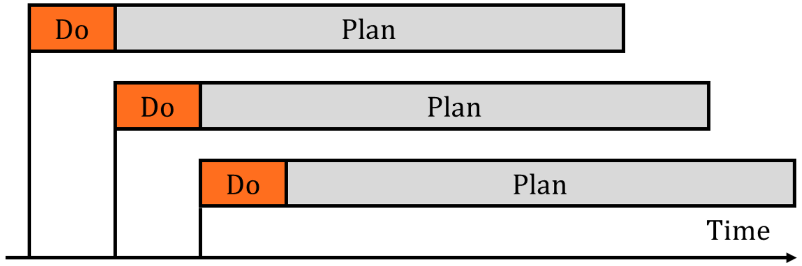
Online Gait Synthesis using MPC





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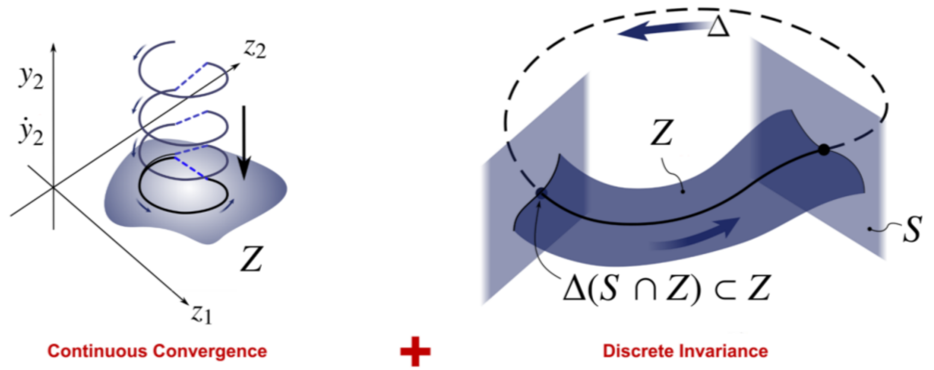

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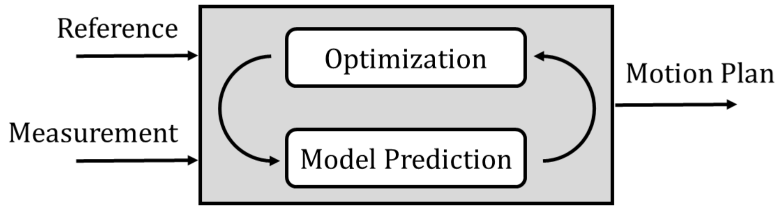

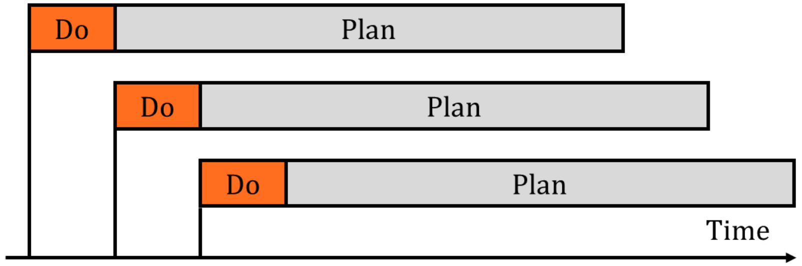

- S-LIP Model, Centroidal Dynamics

A Unified MPC Framework for Whole-Body Dynamic Locomotion and Manipulation, Jean-Pierre Sleiman, 2021

Simplified/Reduced Model Dynamics

Contribution

Contribution

Whole-Body Nonlinear MPC

Contribution

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Reduced computational cost via HZD Reference & Terminal

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Experimental validation on planar biped AMBER-3M

MPC Formulation – Reparametrized Whole-Body Dynamics

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Exclude inverse dynamics from MPC prediciton

- $\boldsymbol{x} = \left(\boldsymbol{q_b}, \boldsymbol{q_j}, \dot{\boldsymbol{q}}_{\boldsymbol{b}}, \dot{\boldsymbol{q}}_{\boldsymbol{j}}\right)^{\boldsymbol{T}}$
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$$\boldsymbol{D}(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} = \boldsymbol{B}\,\boldsymbol{\tau} + \boldsymbol{J}_c^T(\boldsymbol{q})\,\boldsymbol{\lambda}_c$$

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$$\boldsymbol{\tau} = \boldsymbol{J}_c^T\,\boldsymbol{F}_c - \boldsymbol{D}\,\ddot{\boldsymbol{q}} - \boldsymbol{C}\,\dot{\boldsymbol{q}} - \boldsymbol{G}$$

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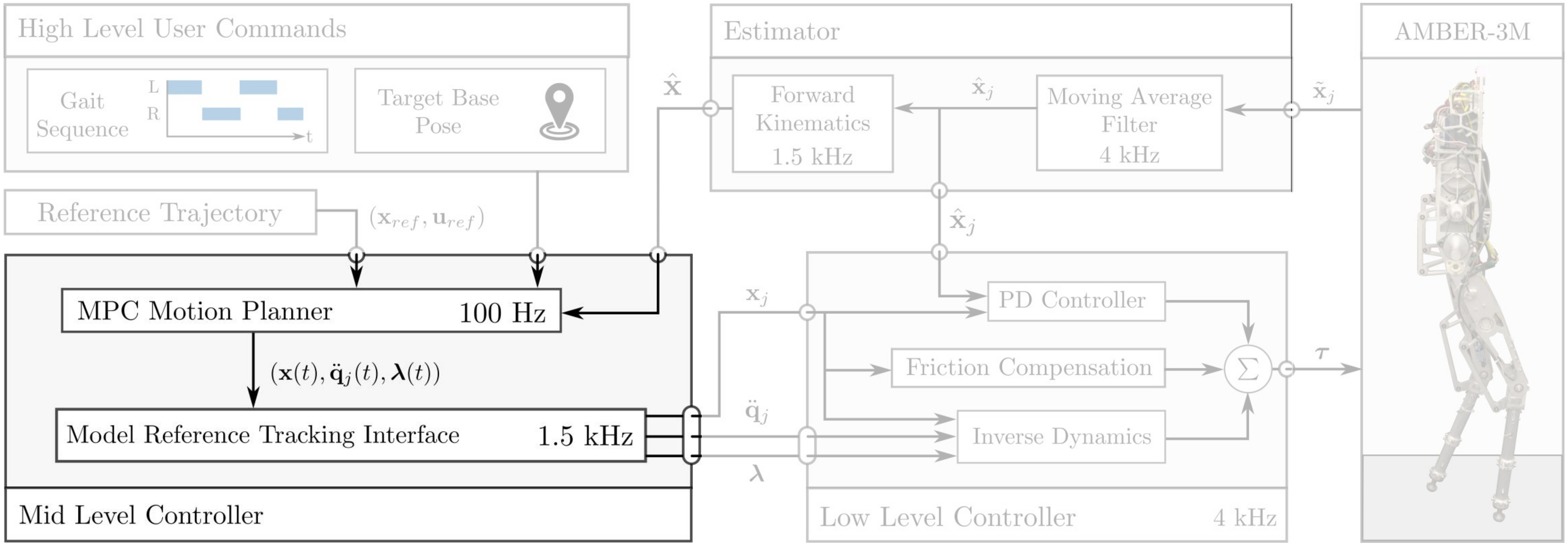
Focus computation on unactuated DoF

Whole-Body nonlinear dynamics/constraints

AMBER Implementation – Control Overview



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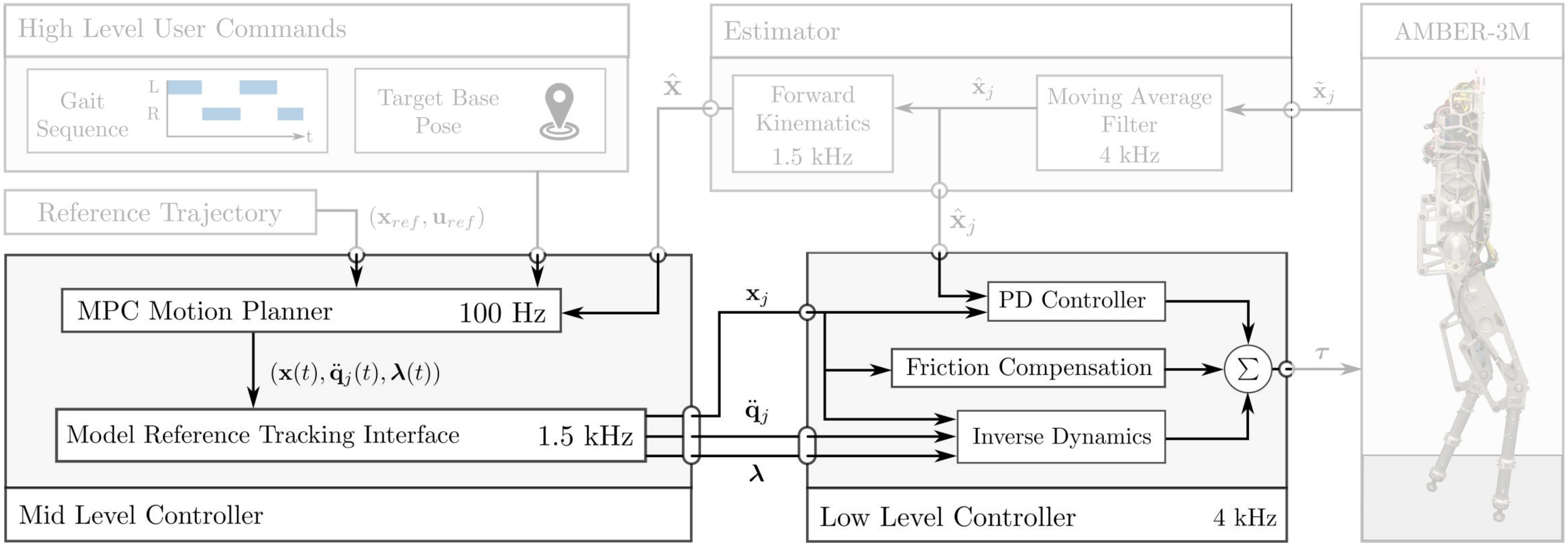

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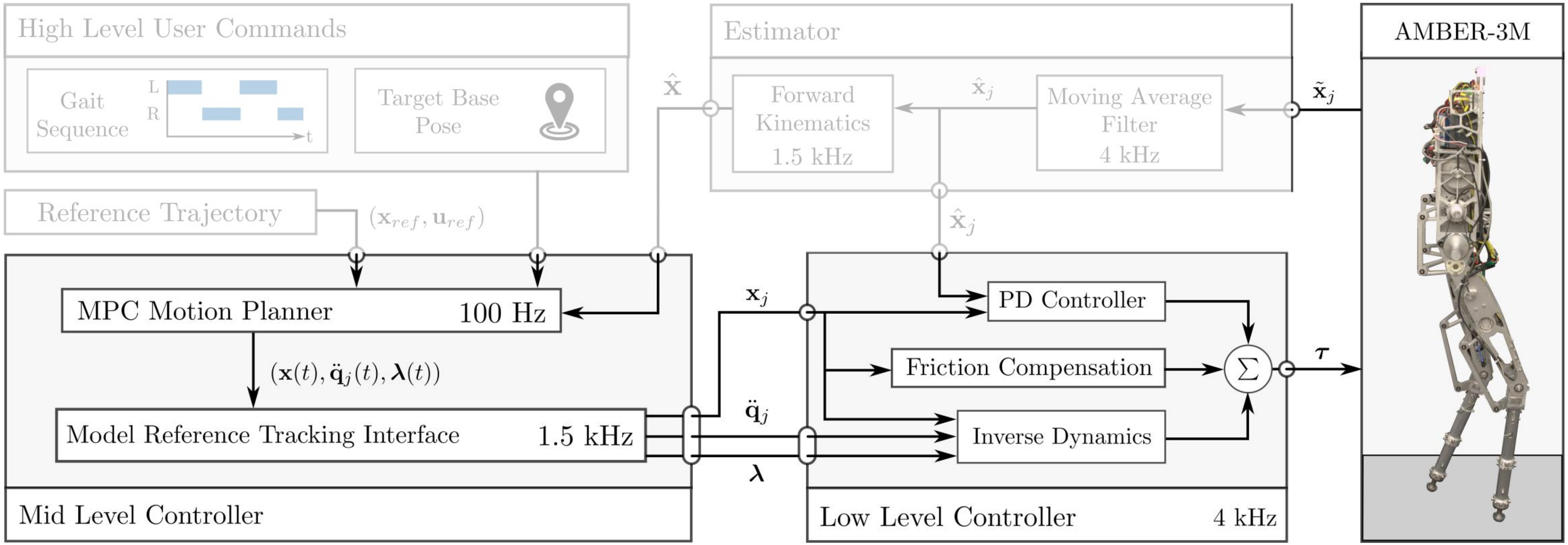

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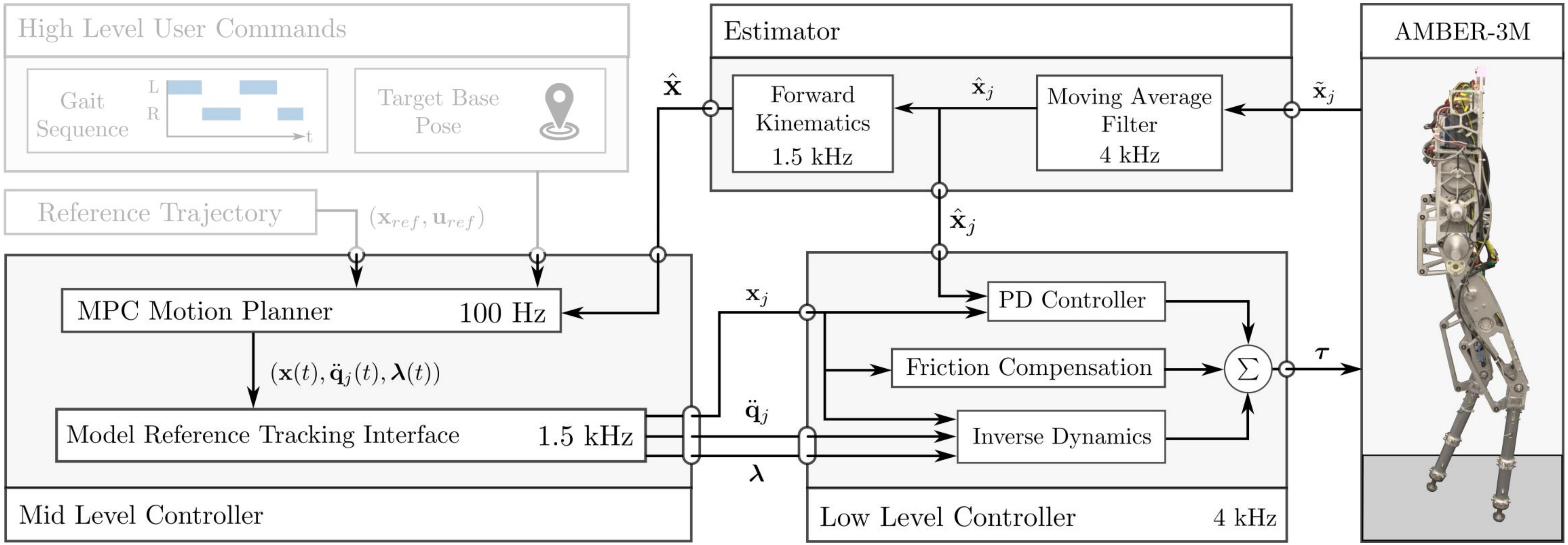

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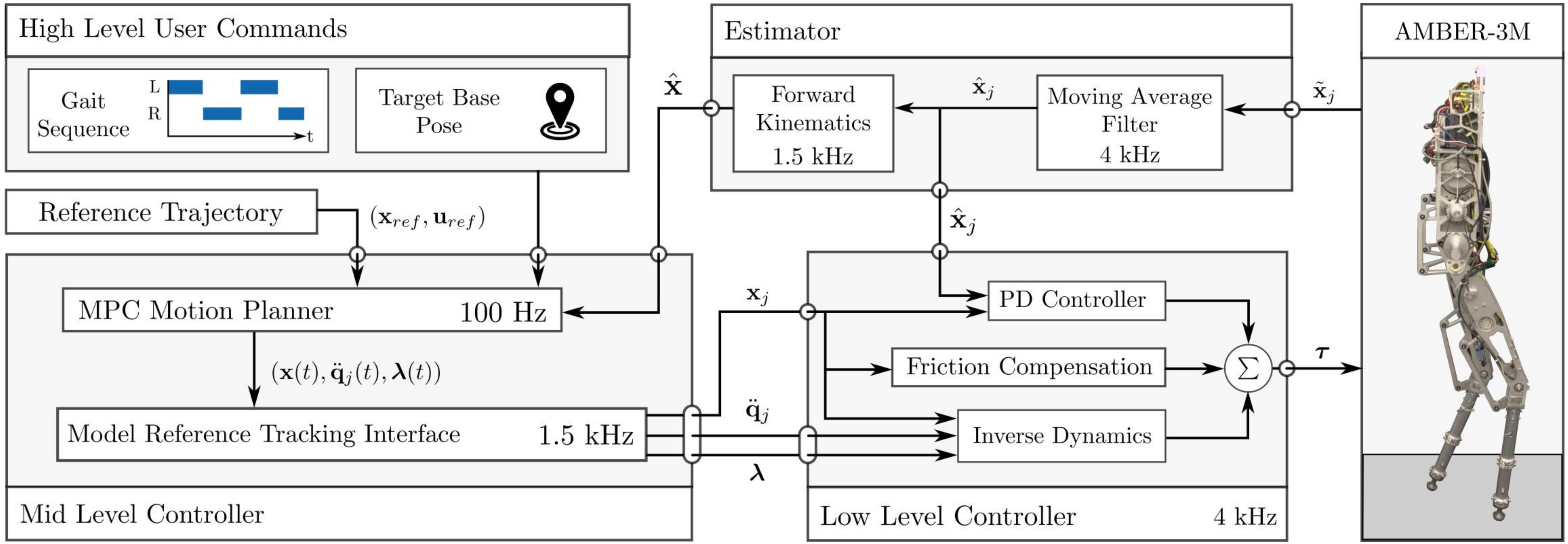
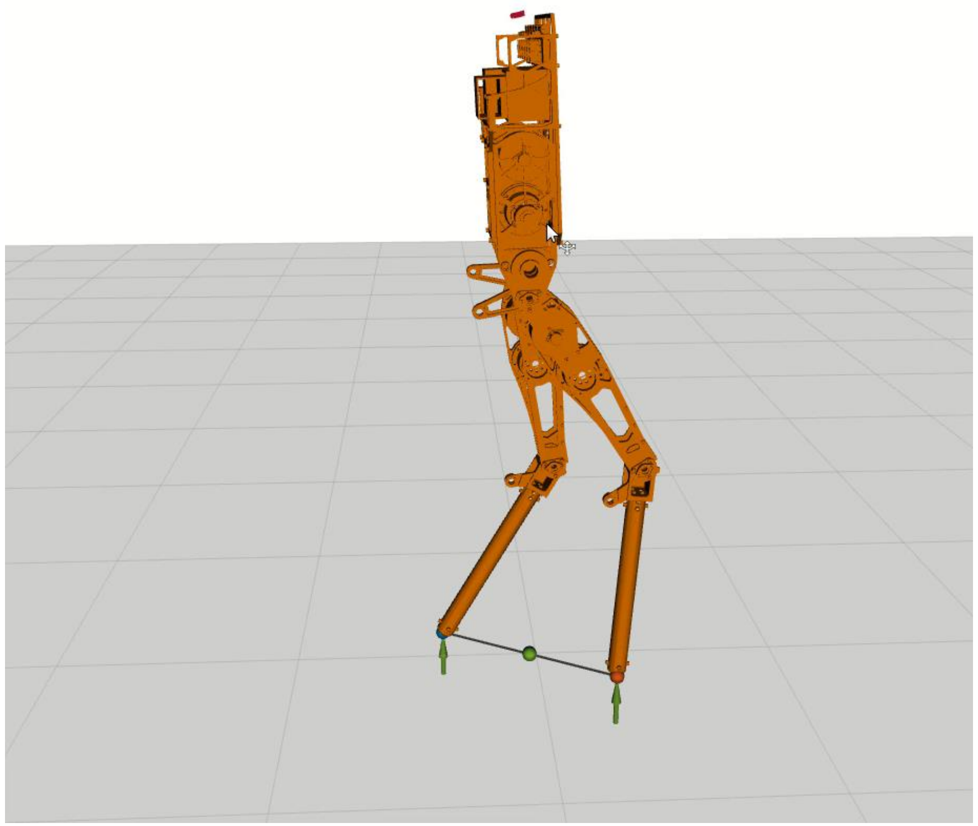
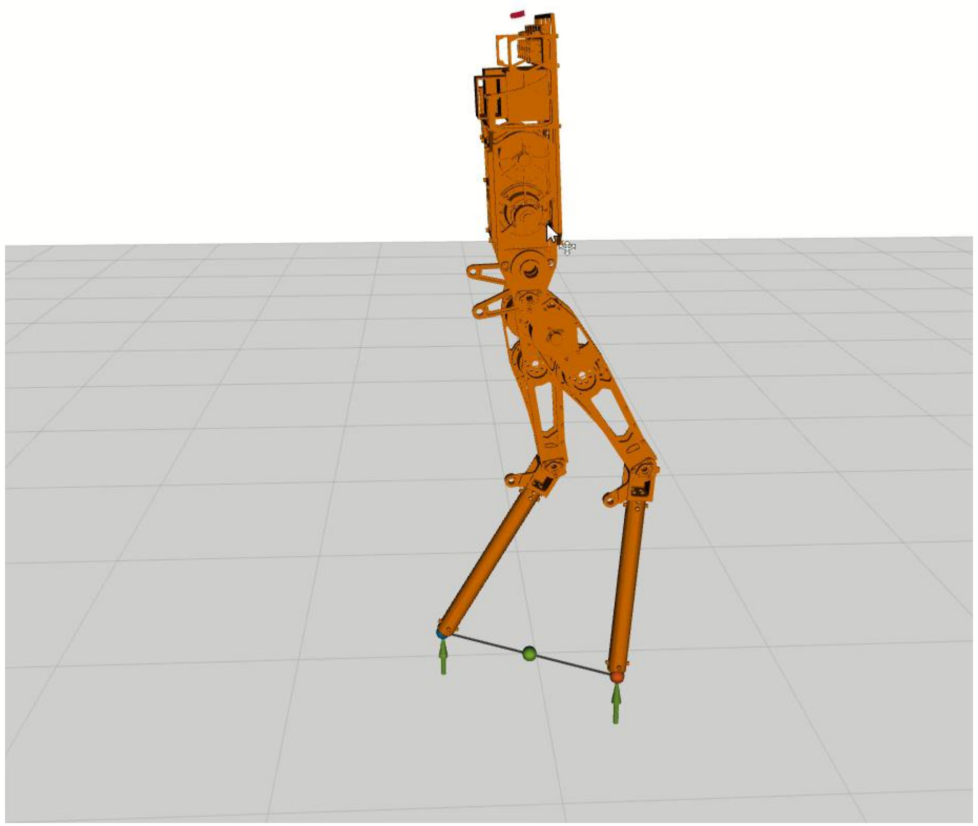
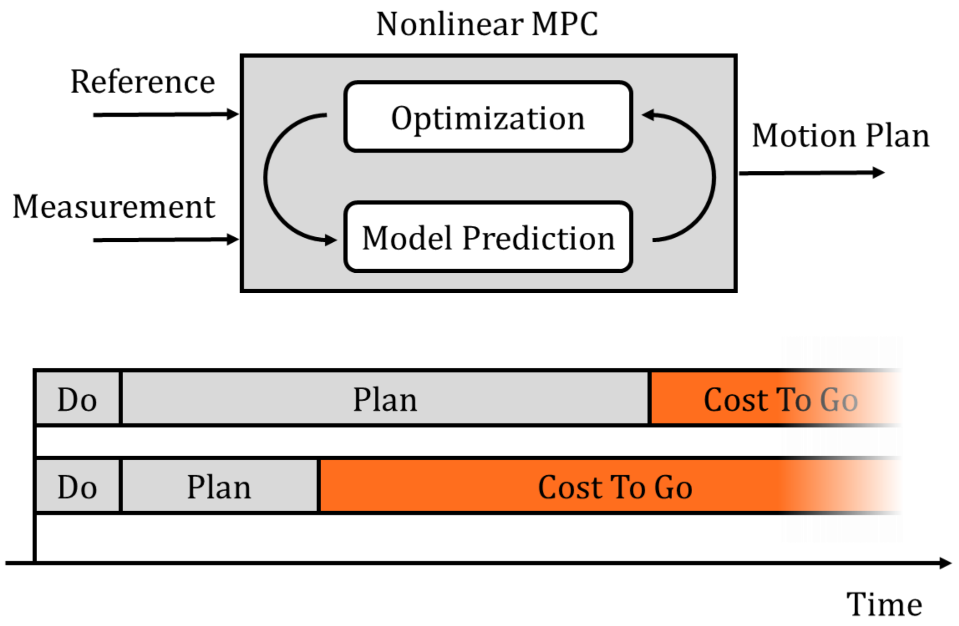


Results - Simulation





Reduce Computational Cost via Horizon Shortening



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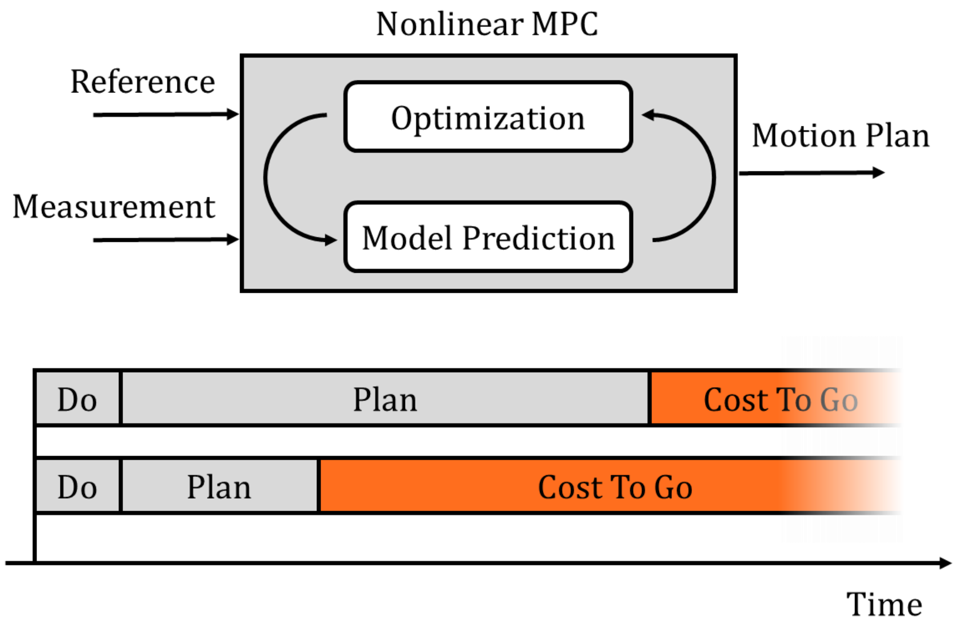

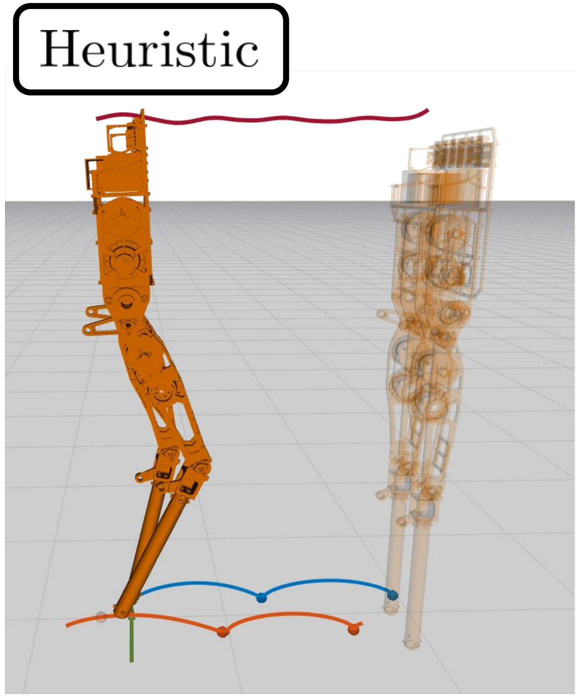

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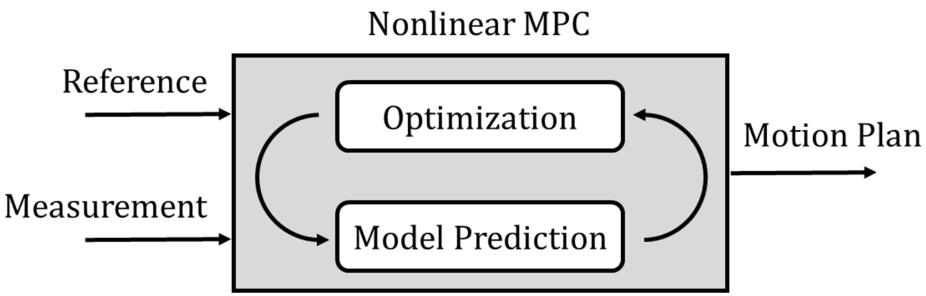
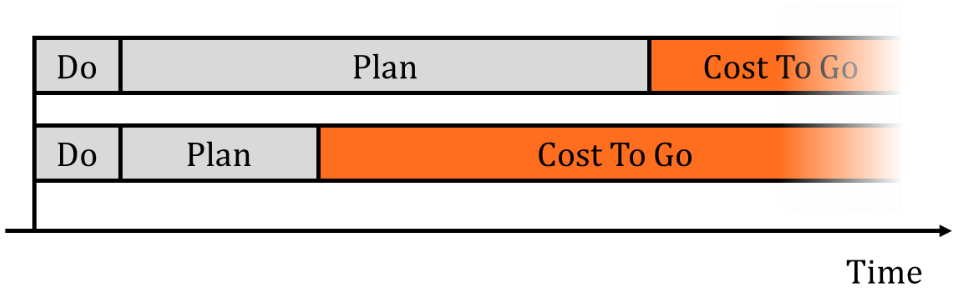
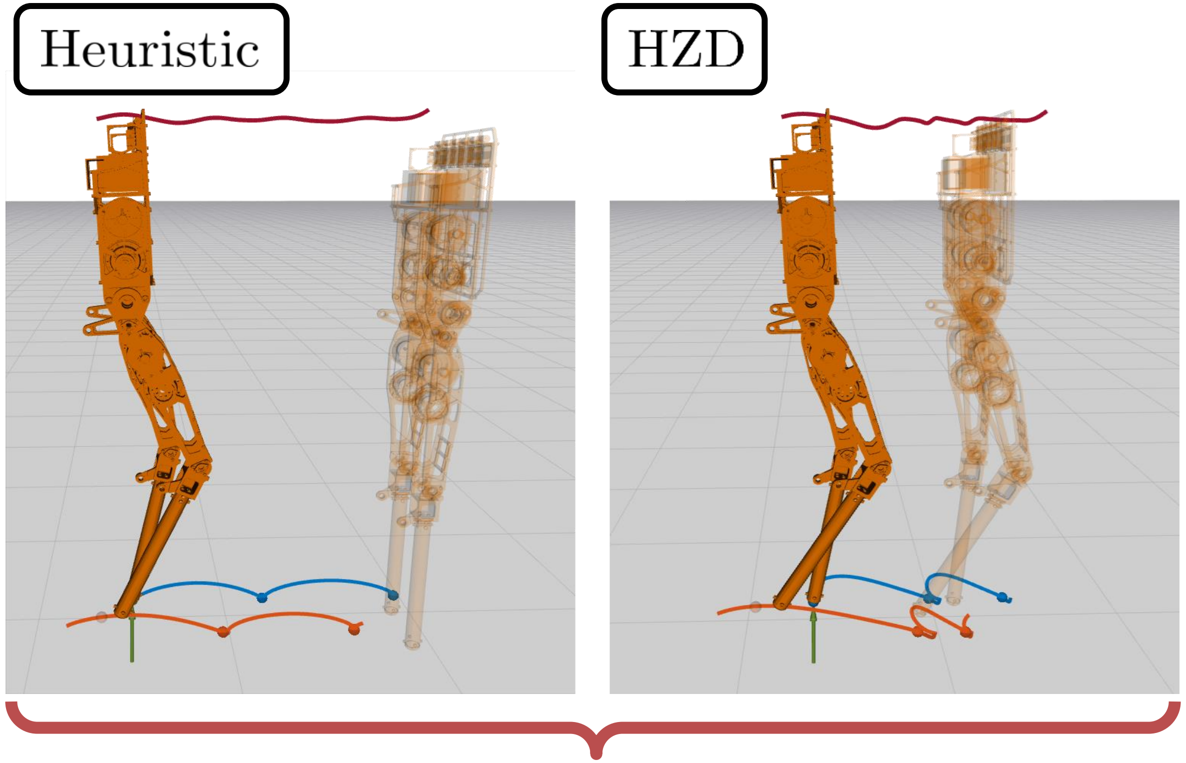


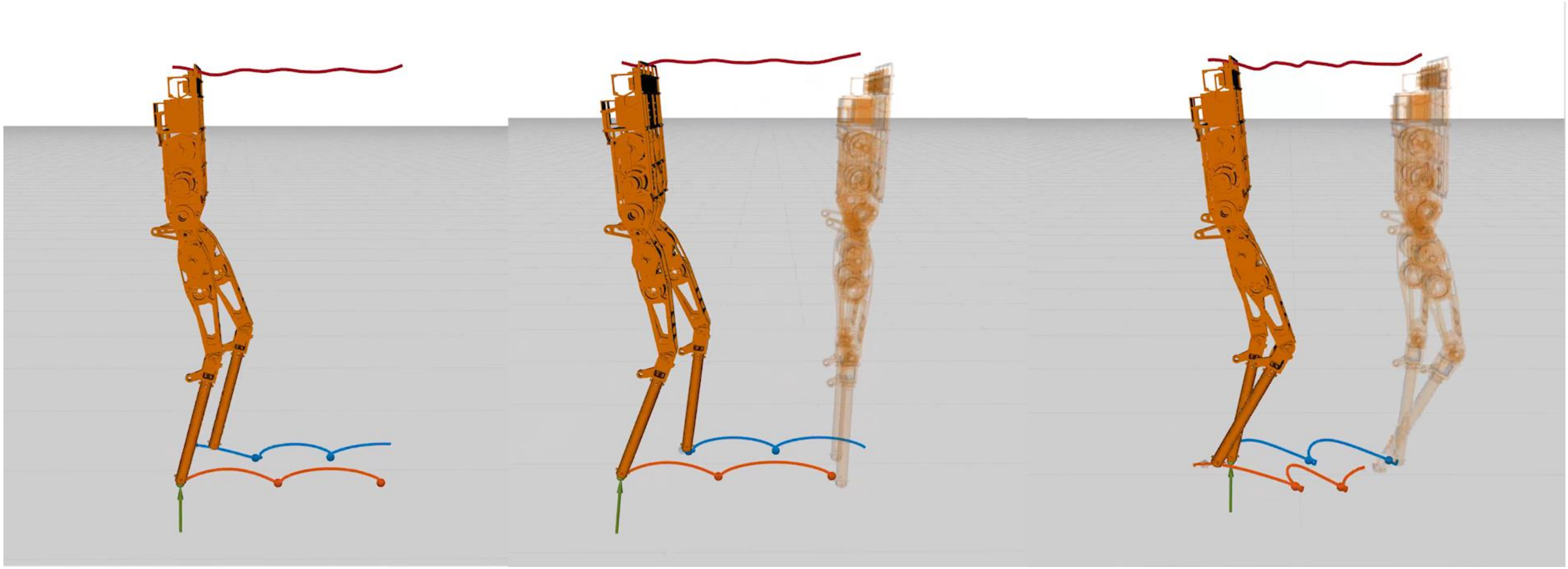




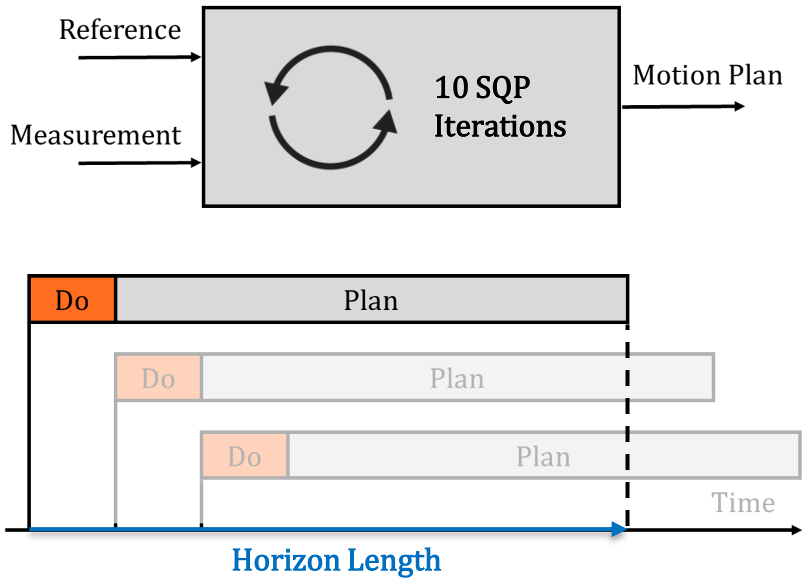
$$\underset{\mathbf{u}(\cdot)}{\text{minimize}} \quad \phi(\mathbf{x}(t_H)) + \int_0^{t_H} l(\mathbf{x}(t), \mathbf{u}(t), t)\mathrm{d}t$$

MPC Terminal Cost Visualization 2s Horizon

No Terminal | Heuristic Terminal | HZD Terminal



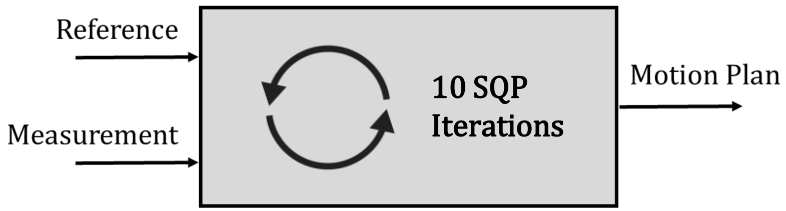
Results - Metrics

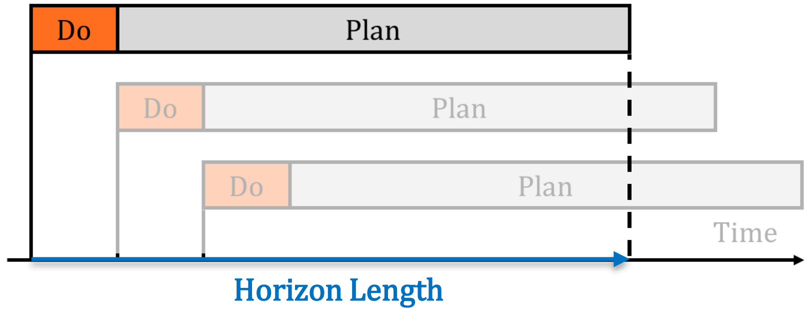


Results - Metrics

Ryzen 9 5950x at 10 SQP Iterations

Horizon Length [s]	2.0	1.0	0.5	0.2
MPC Frequency [Hz]	270	480	670	850





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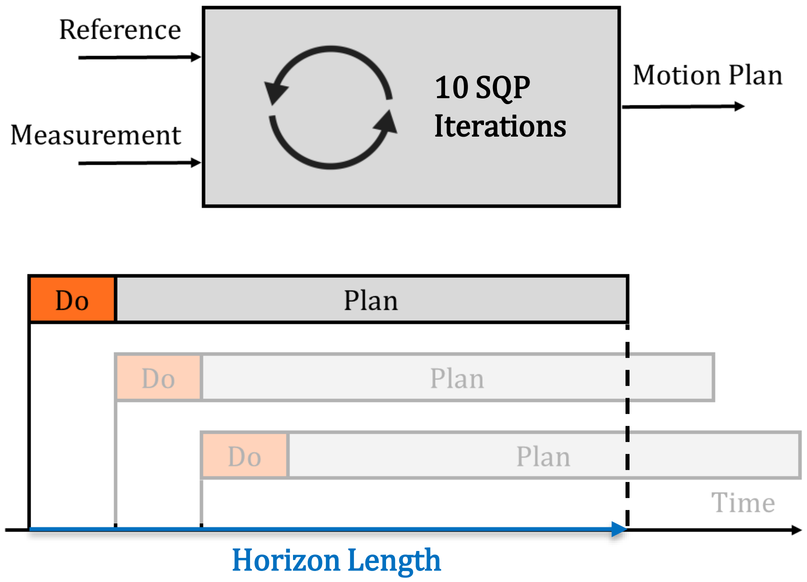
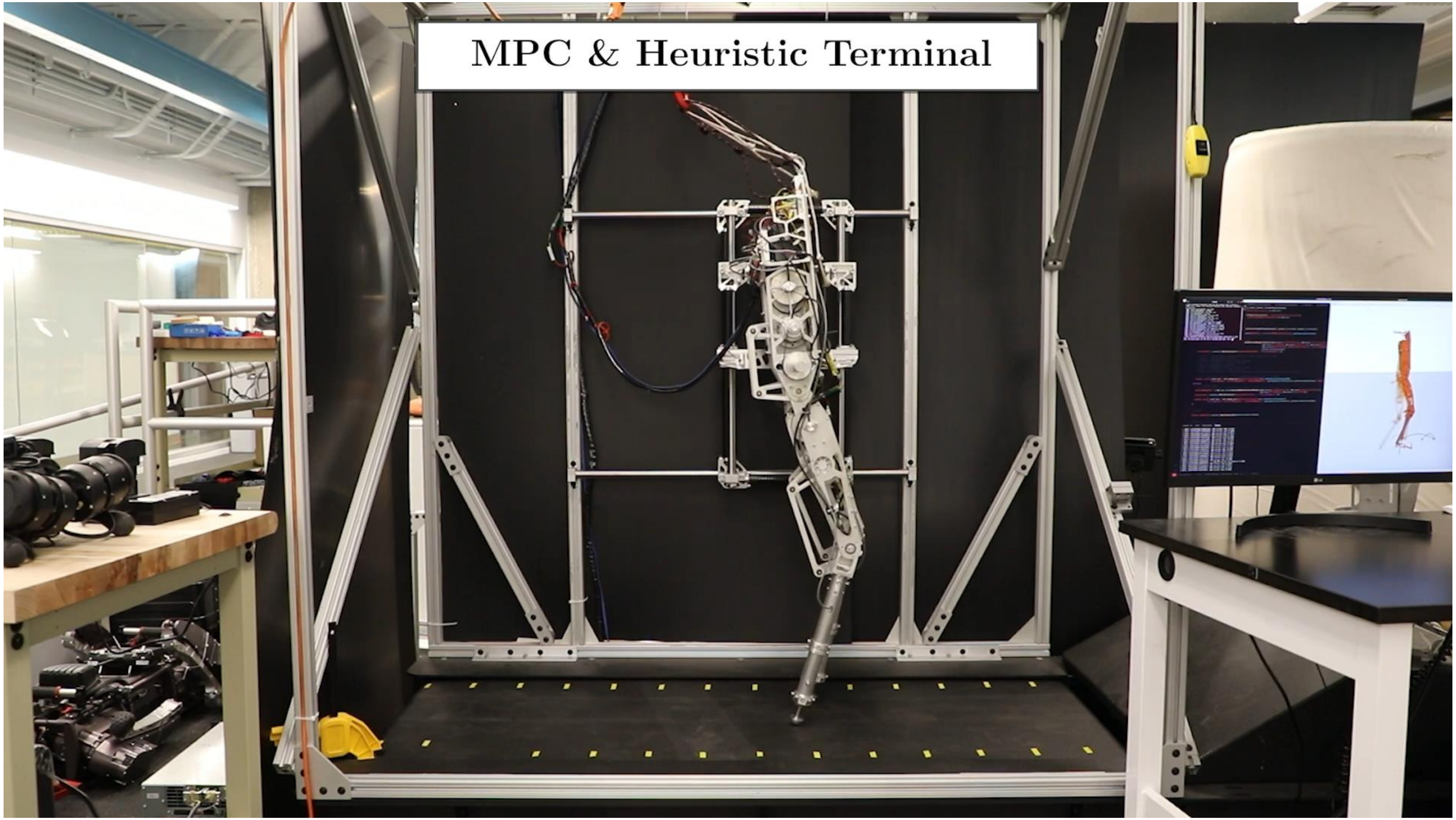
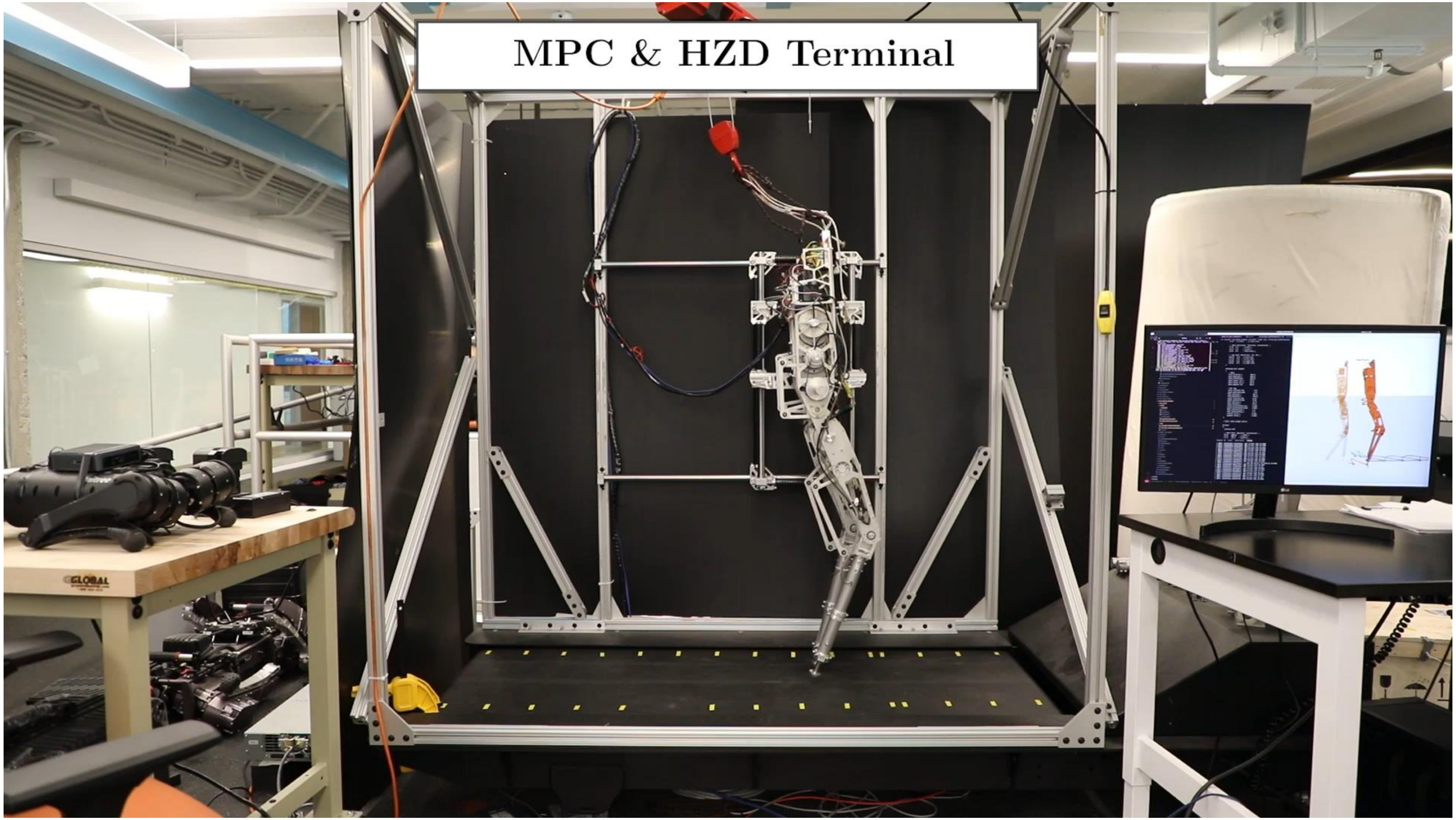


Reduce computational complexity through terminal

Results



Results – MPC & HZD Terminal



Conclusion

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Reparametrized Whole-Body NMPC Formulation

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Significant Horizon shortening through HZD Terminal

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Hardware Demonstration of Whole-Body Online Planning

Thank you for your Attention!

Noel Csomay-Shanklin, AMBER-lab, Caltech

Ruben Grandia, RSL, ETH Zurich

Andrew J. Taylor, AMBER-lab, Caltech

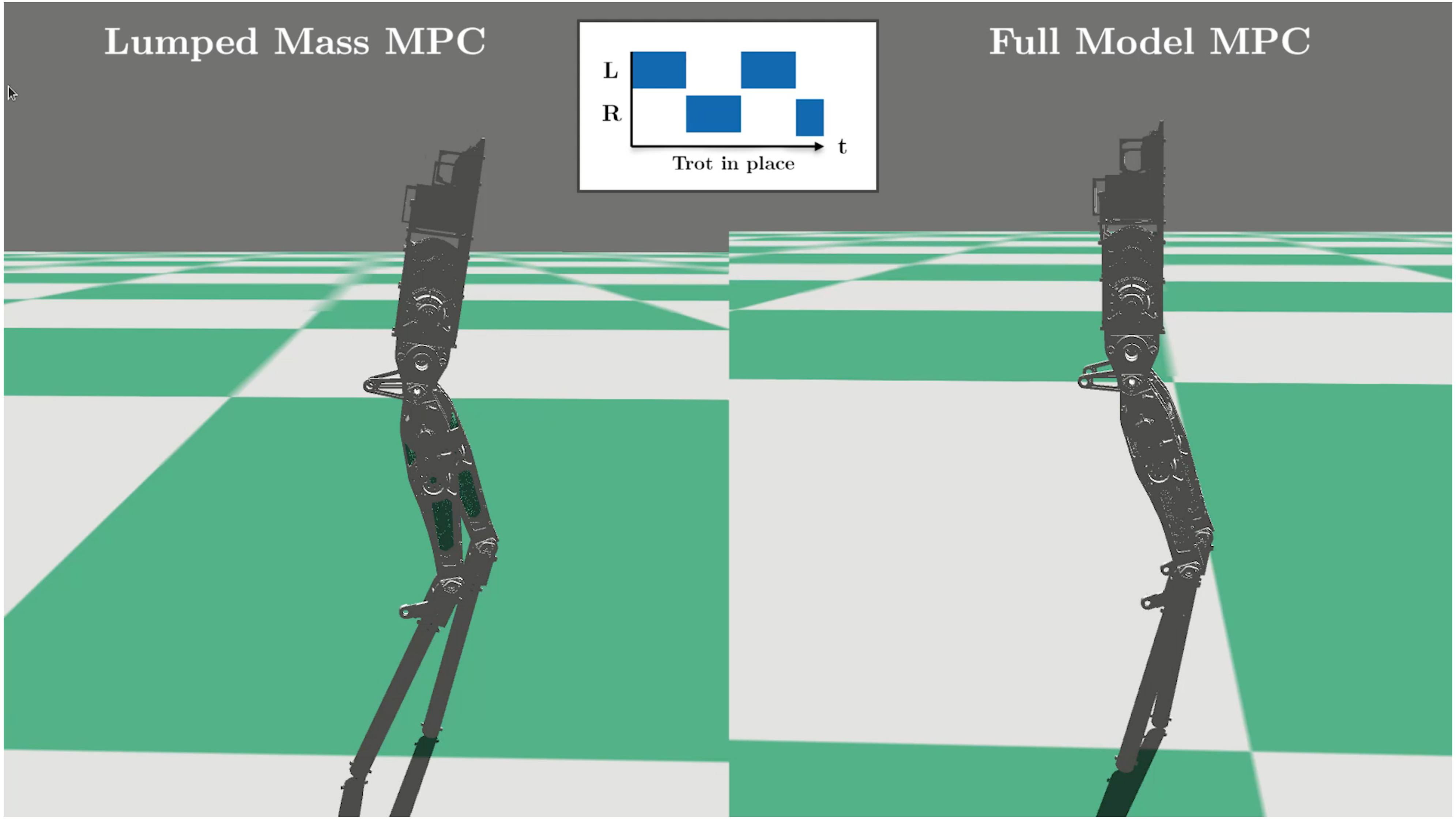
Dr. Farbod Farshidian, RSL, ETH Zurich

Prof. Dr. Aaron Ames, AMBER-lab, Caltech

Prof. Dr. Marco Hutter, RSL, ETH Zurich

Questions?

Results – Comparison to Lumped Mass Model



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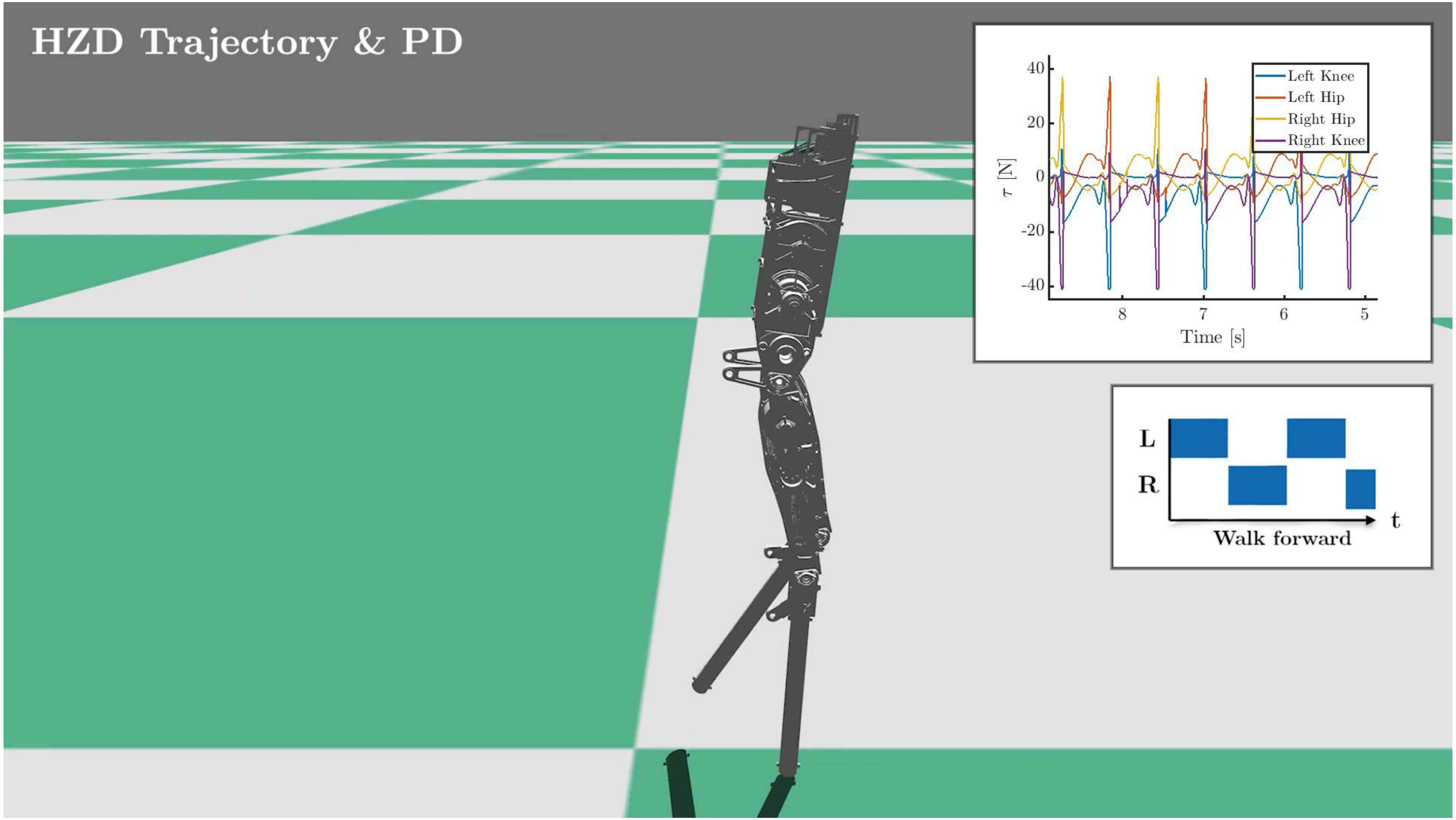

Outlook

- Add impact maps to MPC formulation
- Investigate theoretical properties of using HZD terminal components
- Full computational comparison between centroidal MPC, whole-body MPC and the proposed reparametriced whole-body MPC.
- Transfer approach to 3D bipedal platform

Goal

Dynamic, stable and robust locomotion

- Wide range of behaviors
- Diverse environments

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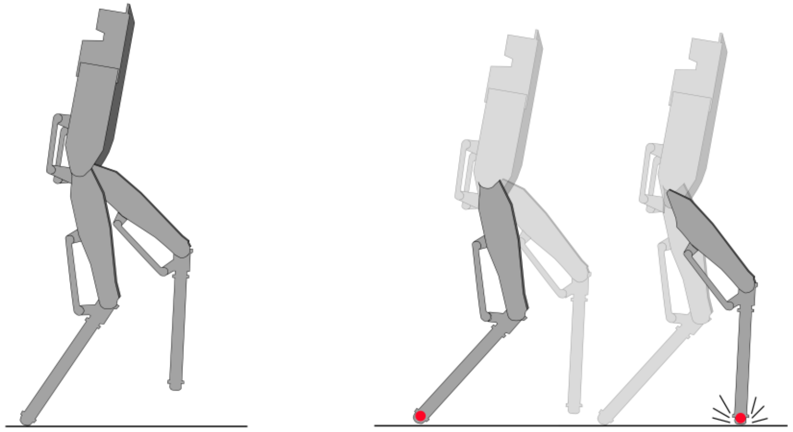
Hybrid Dynamics Model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \qquad \mathbf{x}^{+} = \boldsymbol{\Delta}(\mathbf{x}^{-})$$



Continuous　　Discrete

Legged Robot Dynamics

- Hybrid
- Nonlinear
- Underactuated