Multi-Rate Planning and Control of Uncertain Nonlinear Systems: Model Predictive Control and Control Lyapunov Functions

Noel Csomay-Shanklin†, Andrew J. Taylor†, Ugo Rosolia, Aaron D. Ames

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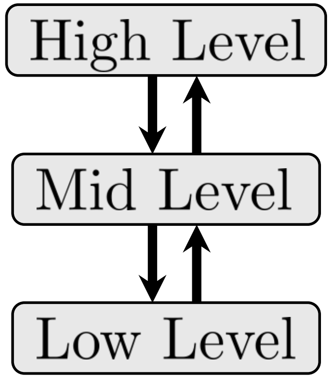
Motivation

(General) Multi-Rate Control:

– Control algorithms are often implemented hierarchically



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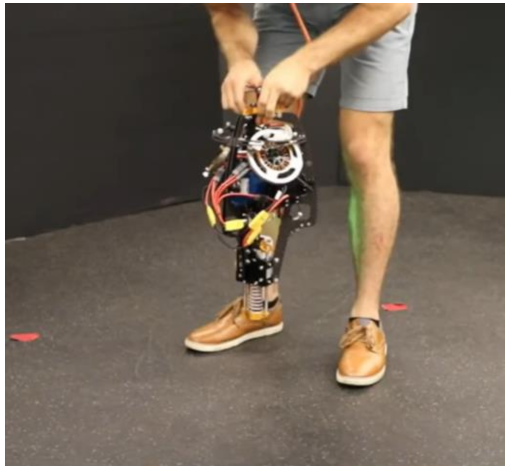

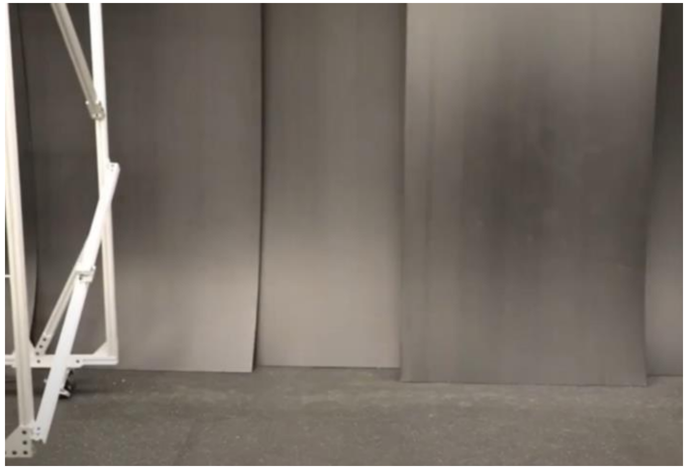

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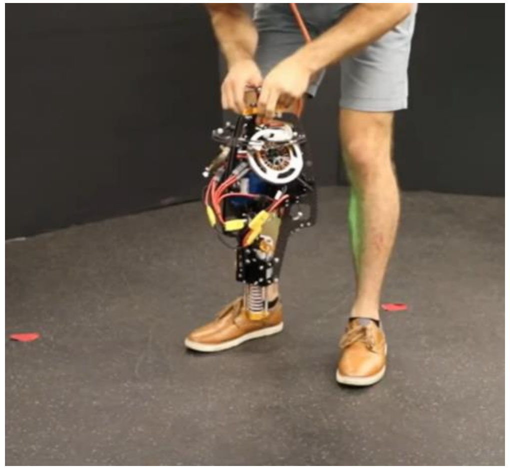

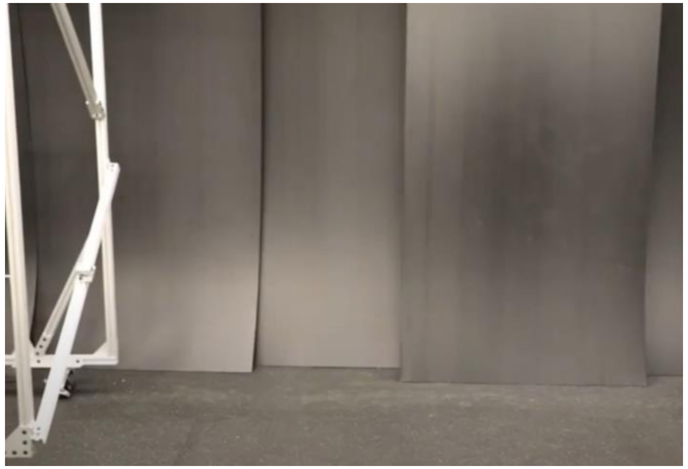

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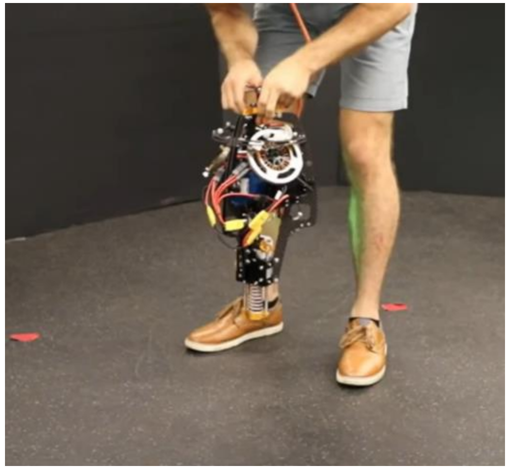
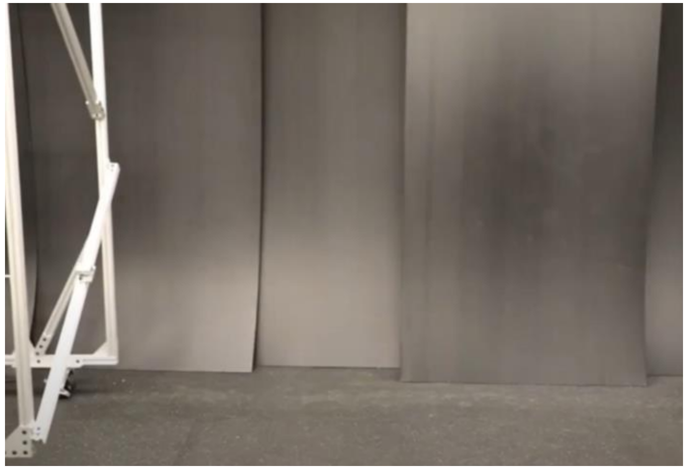
- Need understanding of interactions between levels of hierarchy





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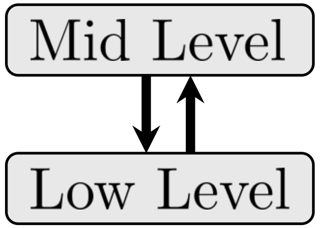

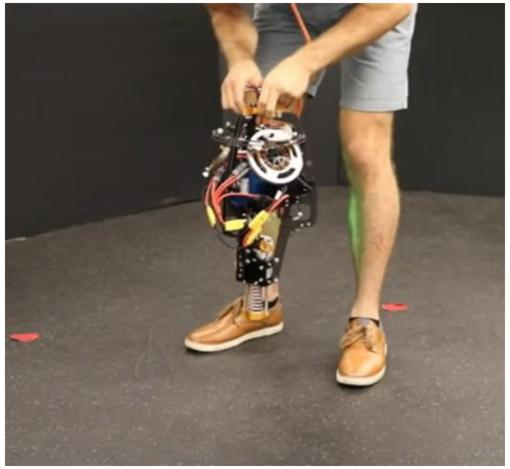

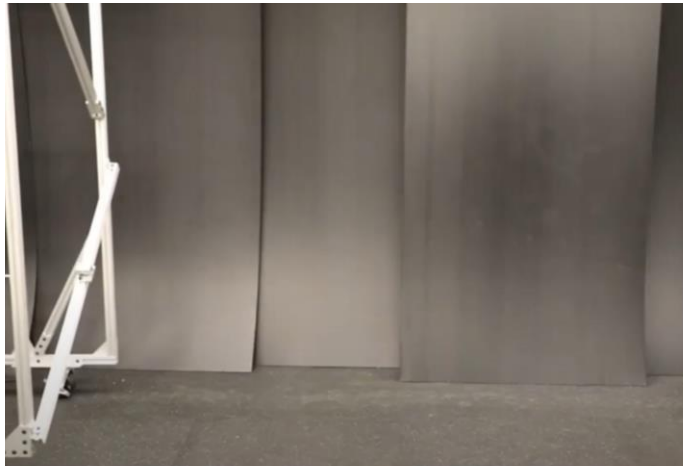

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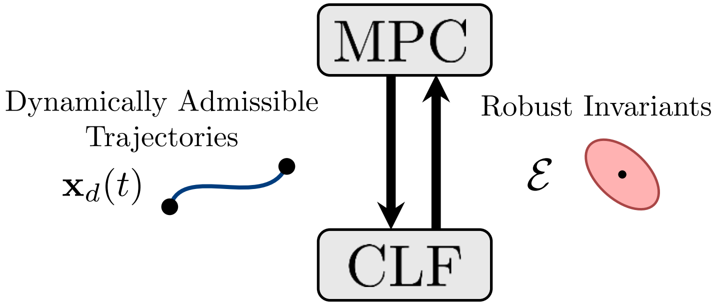

(Low Level) Control Lyapunov Functions (CLFs):

Pros:

- Guarantees of robust stability for nonlinear systems

Cons:

- Myopic decision making
- Difficult to integrate with state and input constraints

(Mid Level) Model Predictive Control (MPC):

Pros:

- Optimality over a horizon
- Naturally incorporates state and input constraints

Cons:

- Computational limits necessitate model approximations

Overview

- Designing a Feedback Controller
- Producing Trajectories and Satisfying State Constraints
- Satisfying Input Constraints
- Multi-Rate Architechture

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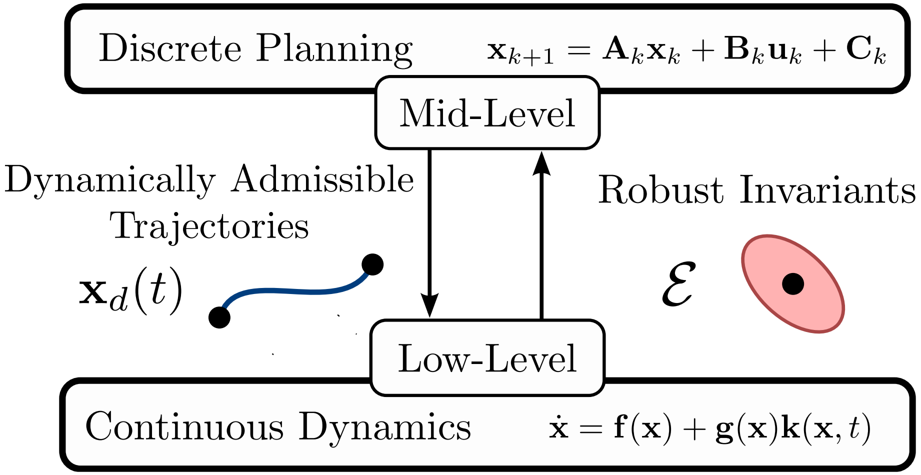
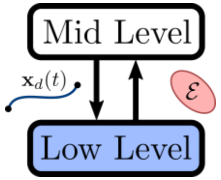


Designing a Feedback Controller



$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ 0 & \mathbf{0}^\top \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{0} \\ f(\mathbf{x}) \end{bmatrix}}_{\mathbf{f}(\mathbf{x})} + \underbrace{\begin{bmatrix} \mathbf{0} \\ g(\mathbf{x}) \end{bmatrix}}_{\mathbf{g}(\mathbf{x})} u + \mathbf{w}(t),$$

State: $\mathbf{x} \in \mathbb{R}^n$

Input: $u \in \mathbb{R}$

Disturbance: $\mathbf{w} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$

Drift Vector: $f : \mathbb{R}^n \to \mathbb{R}$

Actuation Vector: $g : \mathbb{R}^n \to \mathbb{R}$

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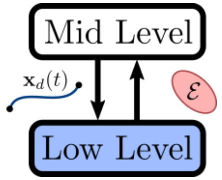

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Assumption 1

– $\mathbf{0} \in \mathbb{R}^n$ is an unforced equilibrium point

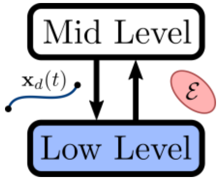
– $g(\mathbf{x}) \neq 0$ for all $\mathbf{x} \in \mathbb{R}^n$

Discussion:

– Satisfied for full state feedback linearizeable systems

– Allows system to follow “arbitrary” trajectories

Designing a Feedback Controller



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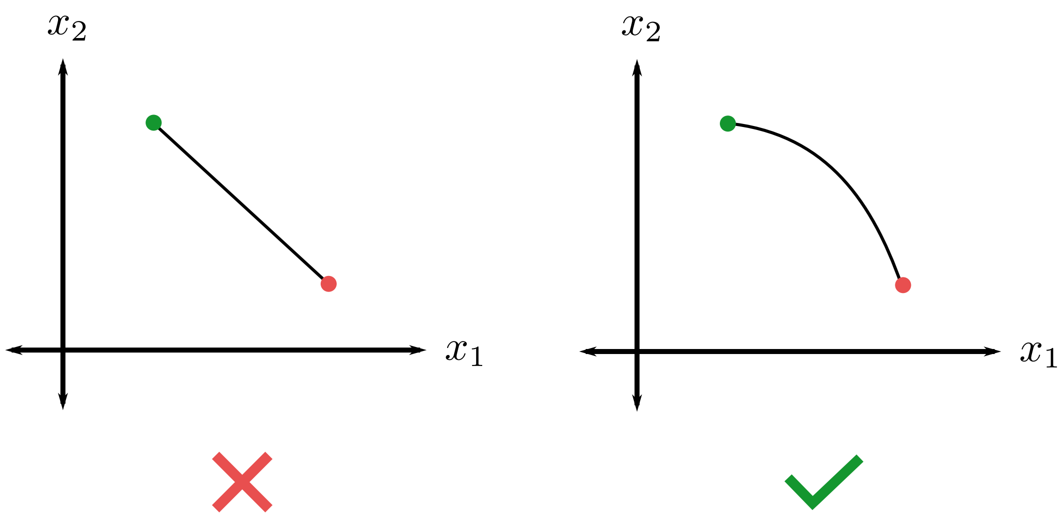


Definition 1 *(Dynamically Admissible Trajectory)*

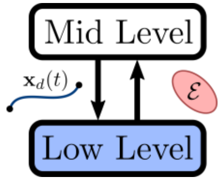
There exists a $u_d : \mathbb{R} \to \mathbb{R}$ such that:

$$\dot{\mathbf{x}}_d(t) = \mathbf{f}(\mathbf{x}_d(t)) + \mathbf{g}(\mathbf{x}_d(t))u_d(t)$$

Discussion:

– $\mathbf{x}_d(t)$ is a “trackable” trajectory

Designing a Feedback Controller



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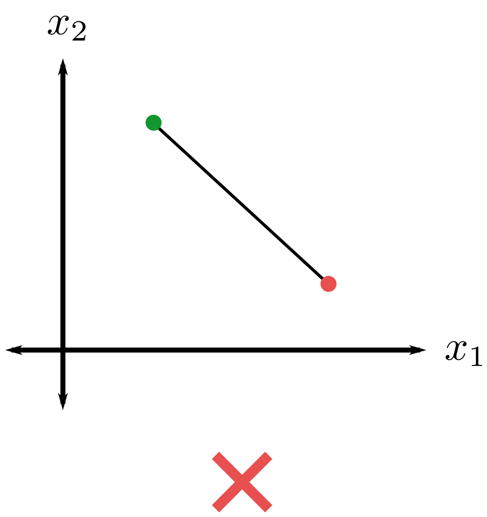

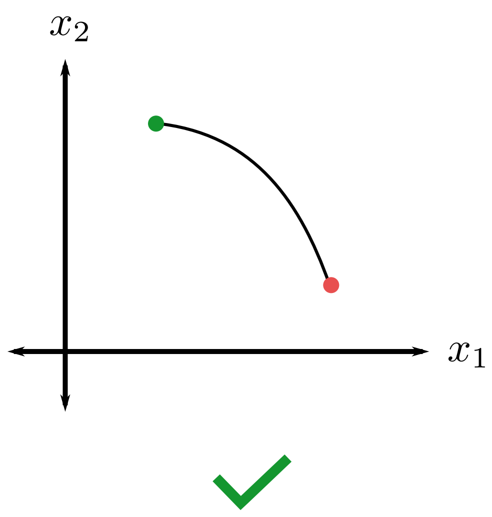

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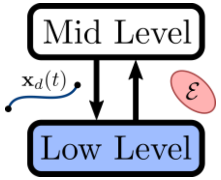
Questions

– How do we synthesize *dynamically admissible trajectories*?

– How to we track them with disturbances?

Designing a Feedback Controller



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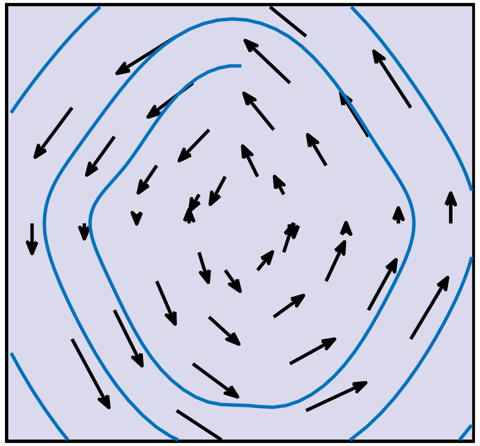

R. Freeman, P. Kokotović, Robust Nonlinear Control Design, 1996.

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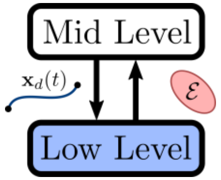

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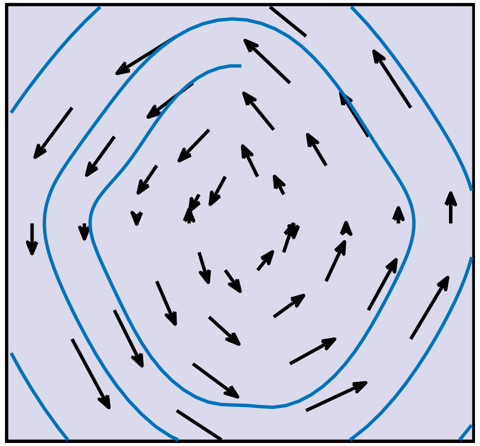

Given a dynamically admissible trajectory $\mathbf{x}_d(t)$, define:

$$\mathbf{e}(\mathbf{x}, t) = \mathbf{x} - \mathbf{x}_d(t)$$

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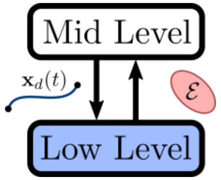

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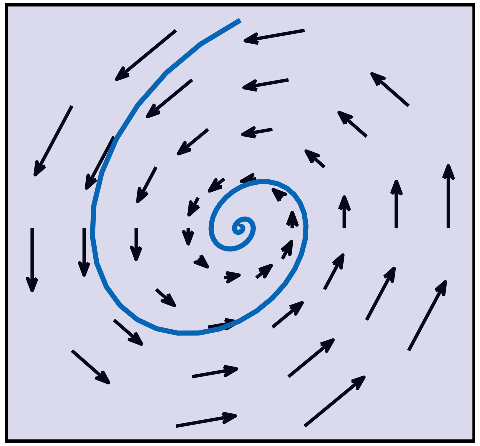

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$$k^{\mathrm{fbl}}(\mathbf{x}, t) = \underbrace{g^{\dagger}(\mathbf{x})(f(\mathbf{x}) - \dot{x}_d^n(t))}_{k^{\mathrm{ff}}(\mathbf{x},t)} - g^{\dagger}(\mathbf{x})\mathbf{K}^{\top}\mathbf{e}(\mathbf{x}, t)$$

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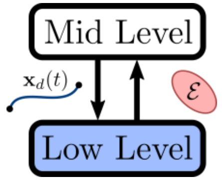

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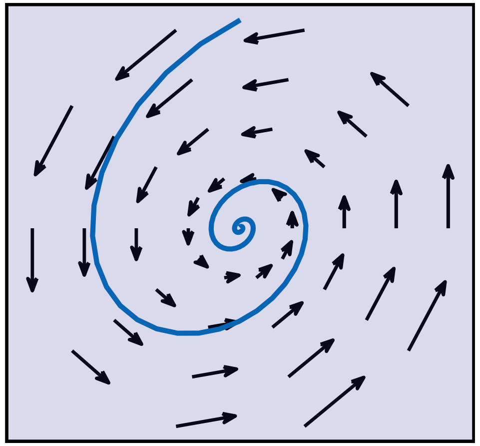

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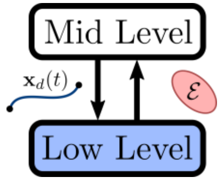

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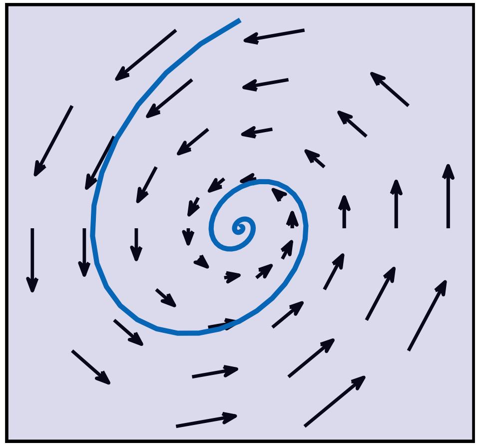

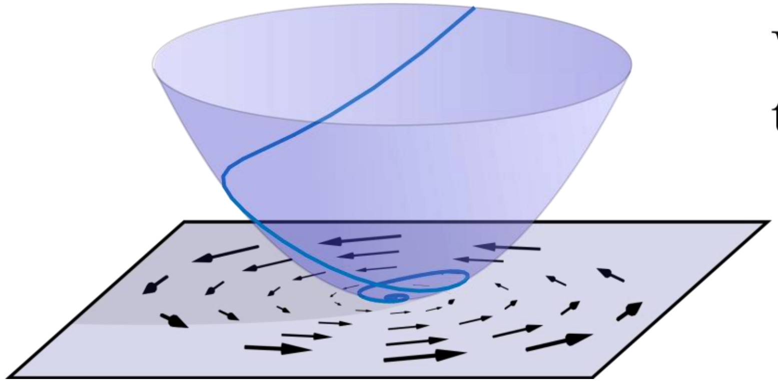

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We can construct a Lyapunov function $V : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ certifying this stability via:

$$V(\mathbf{x},t) = \mathbf{e}(\mathbf{x},t)^{\top}\mathbf{P}\mathbf{e}(\mathbf{x},t)$$

$$\dot{V}(\mathbf{x},t) \leq -\gamma V(\mathbf{x},t)$$

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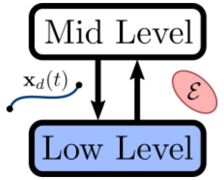

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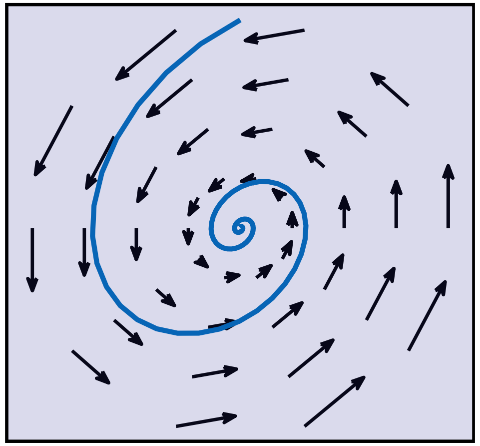

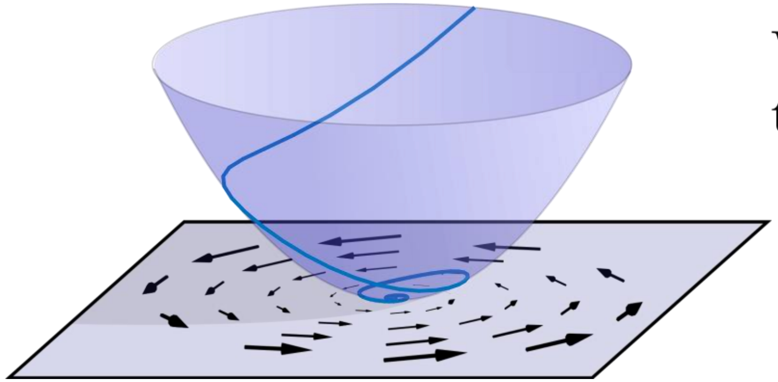

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$$k^{\text{clf}}(\mathbf{x},t) = \underset{u\in\mathbb{R}}{\text{argmin}} \ \frac{1}{2}\|u - k^{\text{ff}}(\mathbf{x},t)\|_2^2 \qquad \text{(CLF-QP)}$$
$$\text{s.t. } \dot{V}(\mathbf{x},u,t) \leq -\gamma V(\mathbf{x},t)$$

A. Ames, M. Powell, Towards the unification of locomotion and manipulation through control lyapunov functions and quadratic programs, 2013.

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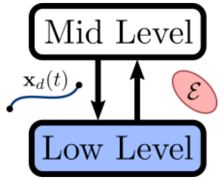

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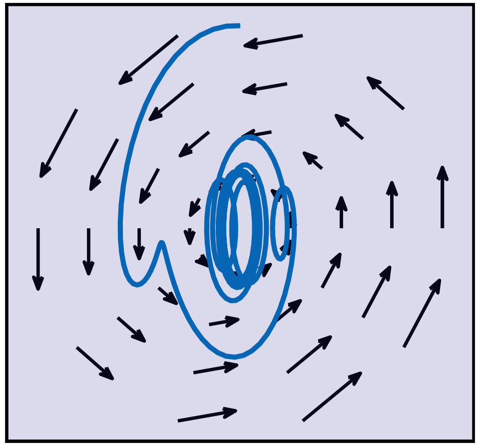

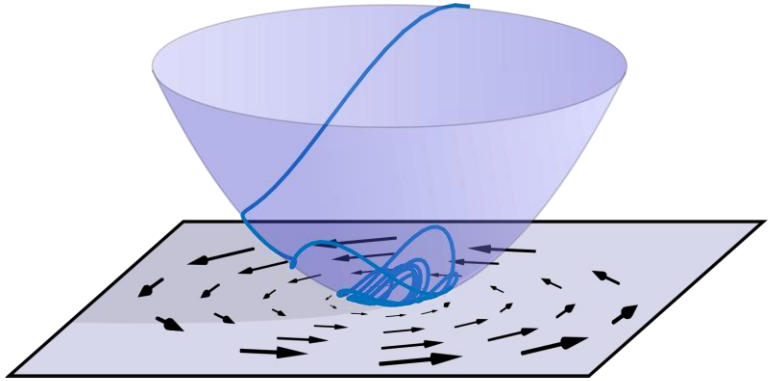

When disturbances are present, we instead have:

$$\dot{\mathbf{e}}(\mathbf{x},t) = \mathbf{F}\mathbf{e}(\mathbf{x},t) + \mathbf{w}(t)$$

Input to State Stability yields some $c(\|\mathbf{w}\|_\infty) \in \mathbb{R}$:

$$V(\mathbf{x},t) \geq c(\|\mathbf{w}\|_\infty) \implies \dot{V}(\mathbf{x},t) \leq -\gamma V(\mathbf{x},t)$$

E. Sontag, Y. Wang, On Characterizations of input-to-state stability with respect to compact sets, 1995

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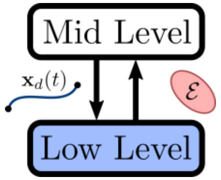

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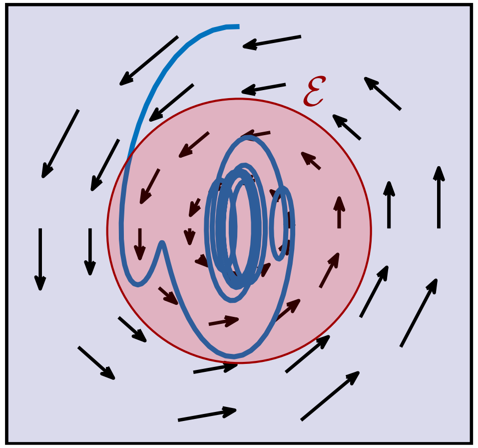

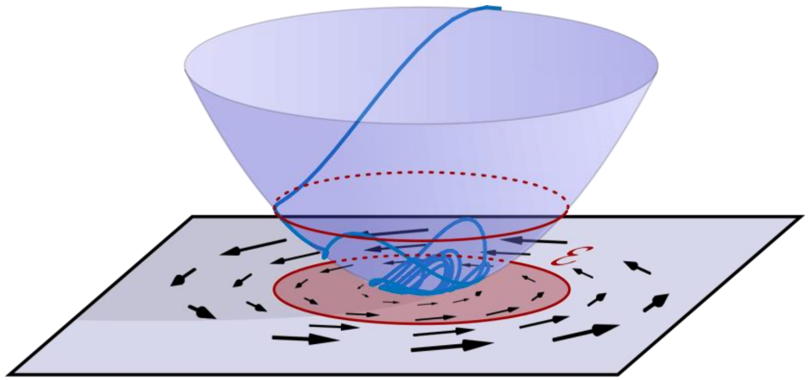

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Lemma 1

Under $k^{\text{clf}}(\mathbf{x},t)$, the solution $\boldsymbol{\varphi}(t) \in \mathcal{E}(t)$, where:

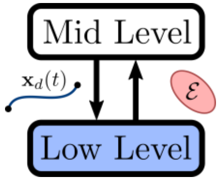
$$\mathcal{E}(t) = \{\mathbf{x} \in \mathbb{R}^n \mid V(\mathbf{x},t) = \mathbf{e}(\mathbf{x},t)^\top \mathbf{P}\mathbf{e}(\mathbf{x},t) \leq c(\|\mathbf{w}\|_\infty)\}$$

Discussion:

- $\mathcal{E}(t)$ is a robust invariant for the nonlinear system

E. Sontag, Y. Wang, On Characterizations of input-to-state stability with respect to compact sets, 1995

Designing a Feedback Controller



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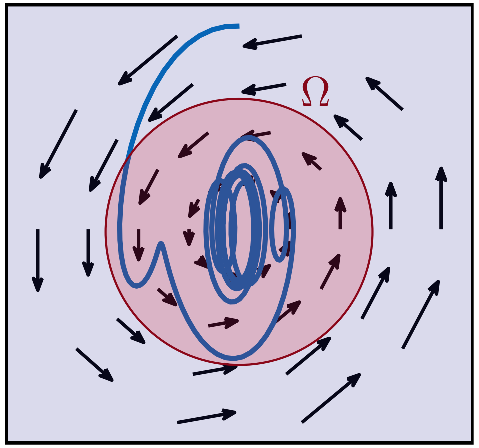
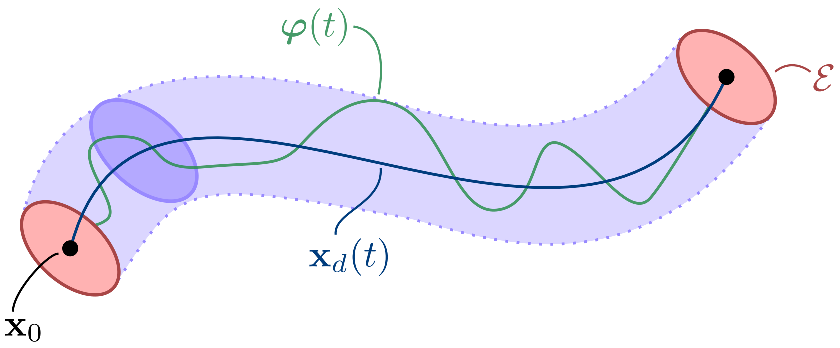




When disturbances are present, we instead have:

$$\dot{\mathbf{e}}(\mathbf{x},t) = \mathbf{F}\mathbf{e}(\mathbf{x},t) + \mathbf{w}(t)$$

Input to State Stability yields some $c(\|\mathbf{w}\|_\infty) \in \mathbb{R}$:

$$V(\mathbf{x},t) \geq c(\|\mathbf{w}\|_\infty) \implies \dot{V}(\mathbf{x},t) \leq -\gamma V(\mathbf{x},t)$$

Lemma 1

Under $k^{\mathrm{clf}}(\mathbf{x},t)$, the solution $\boldsymbol{\varphi}(t) \in \mathcal{E}(t)$, where:

$$\mathcal{E}(t) = \{\mathbf{x} \in \mathbb{R}^n \mid V(\mathbf{x},t) = \mathbf{e}(\mathbf{x},t)^\top \mathbf{P}\mathbf{e}(\mathbf{x},t) \leq c(\|\mathbf{w}\|_\infty)\}$$

Discussion:

– $\mathcal{E}(t)$ is a robust invariant for the nonlinear system

E. Sontag, Y. Wang, On Characterizations of input-to-state stability with respect to compact sets, 1995

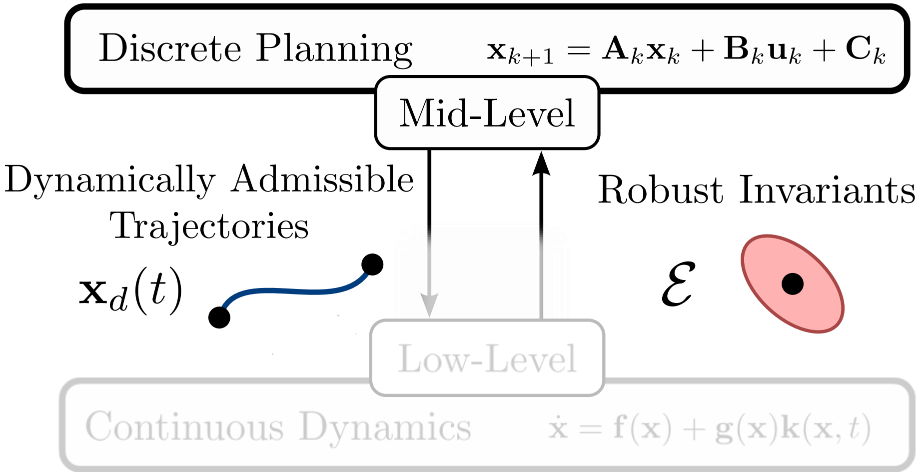
Overview

– Designing a Feedback Controller

– Producing Trajectories and Satisfying State Constraints

– Satisfying Input Constraints

– Multi-Rate Architechture



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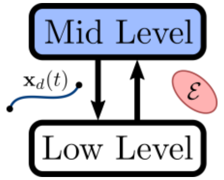
Producing Trajectories and Satisfying State Constraints



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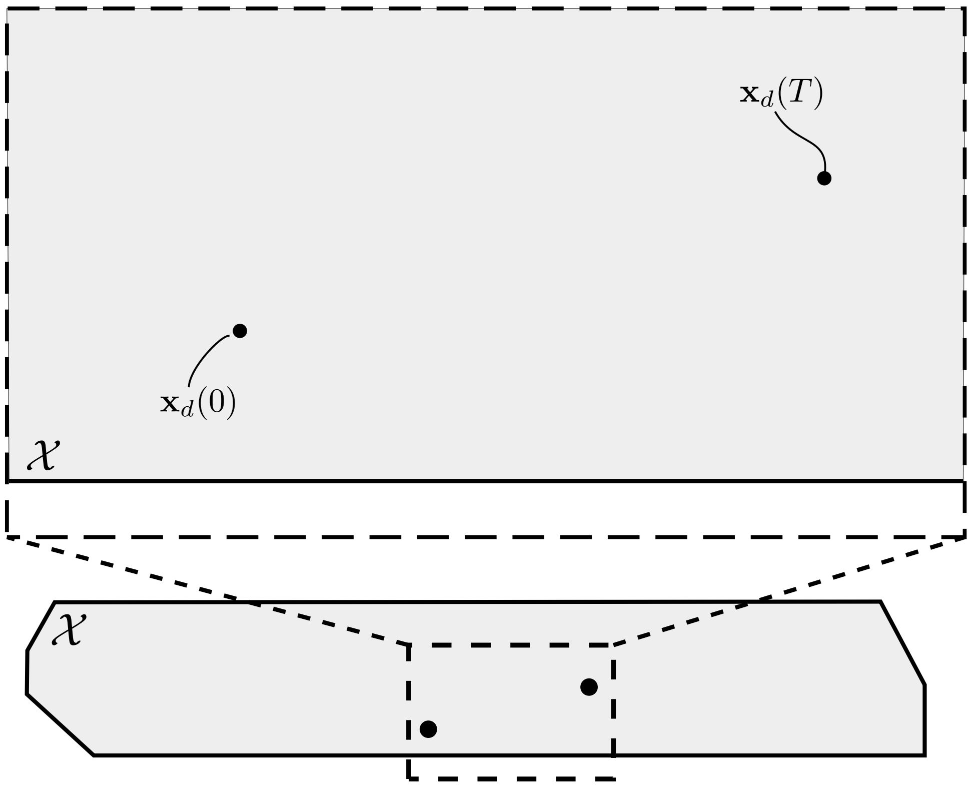
Approach: $\mathbf{x}_d(t) \oplus \mathcal{E} \in \mathcal{X}$



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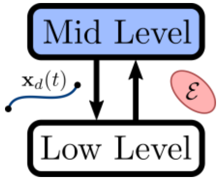

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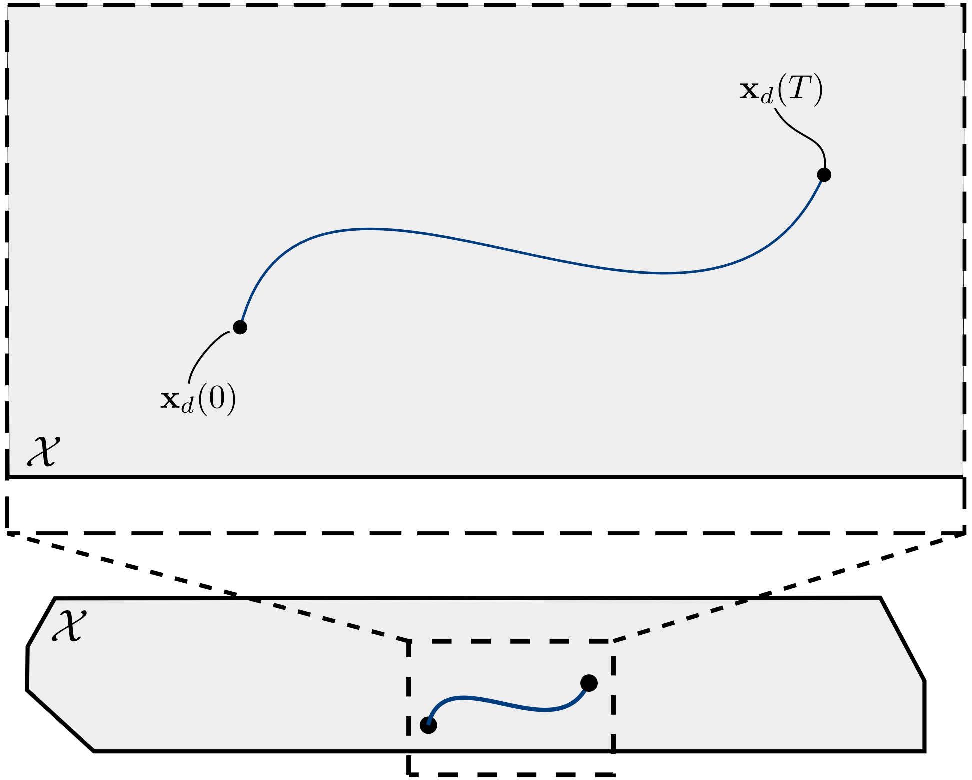

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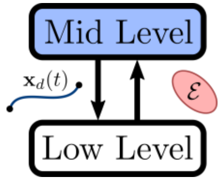

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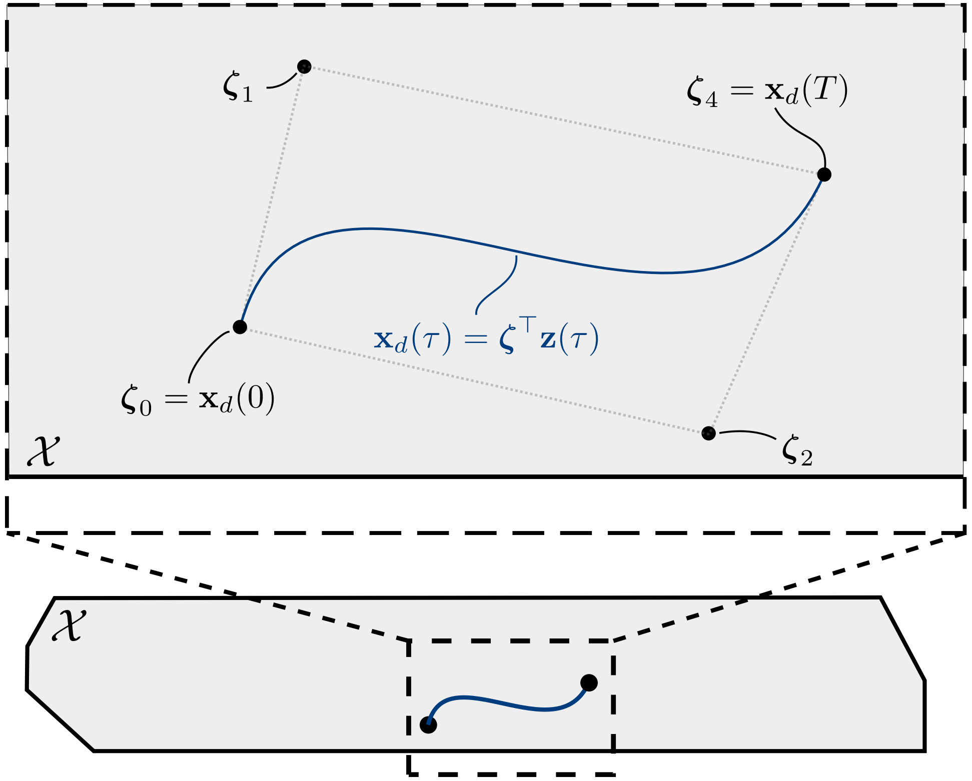

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M. Kamermans, A primer on bézier curves

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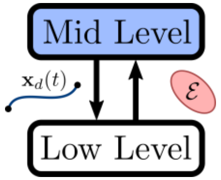

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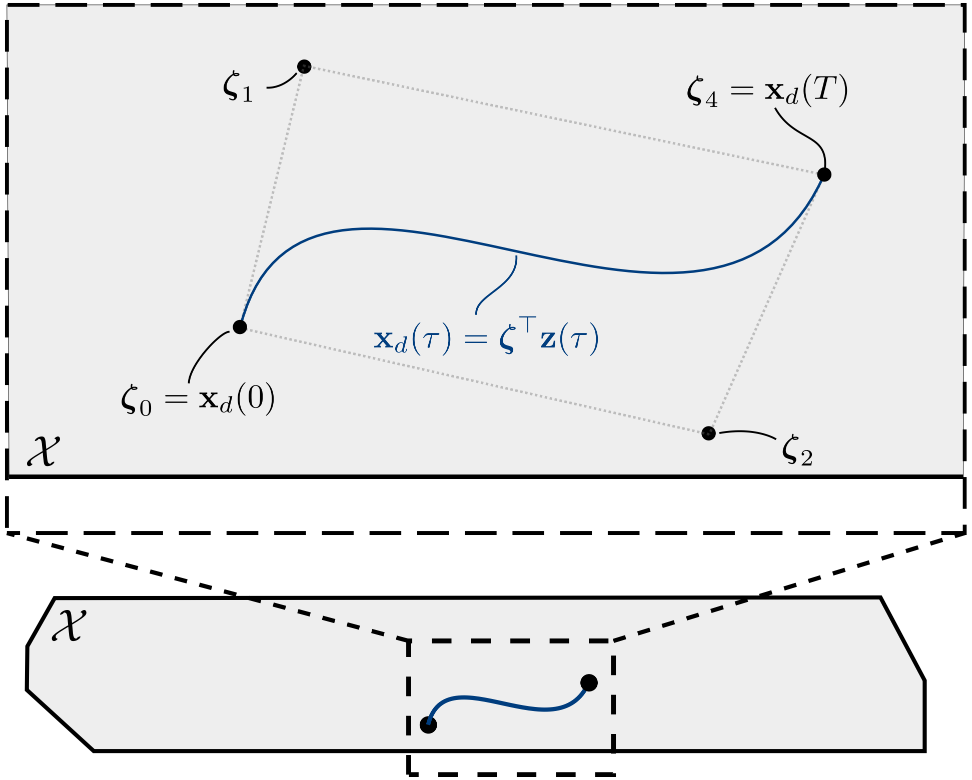

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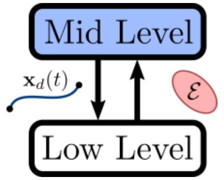

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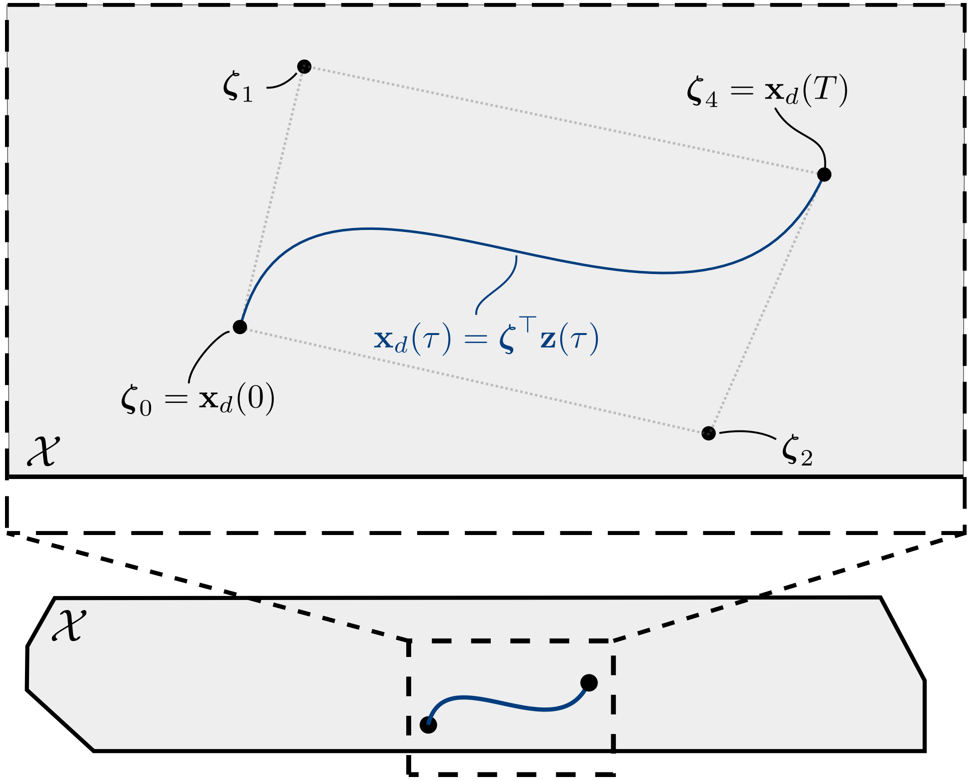

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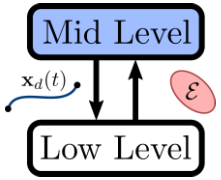

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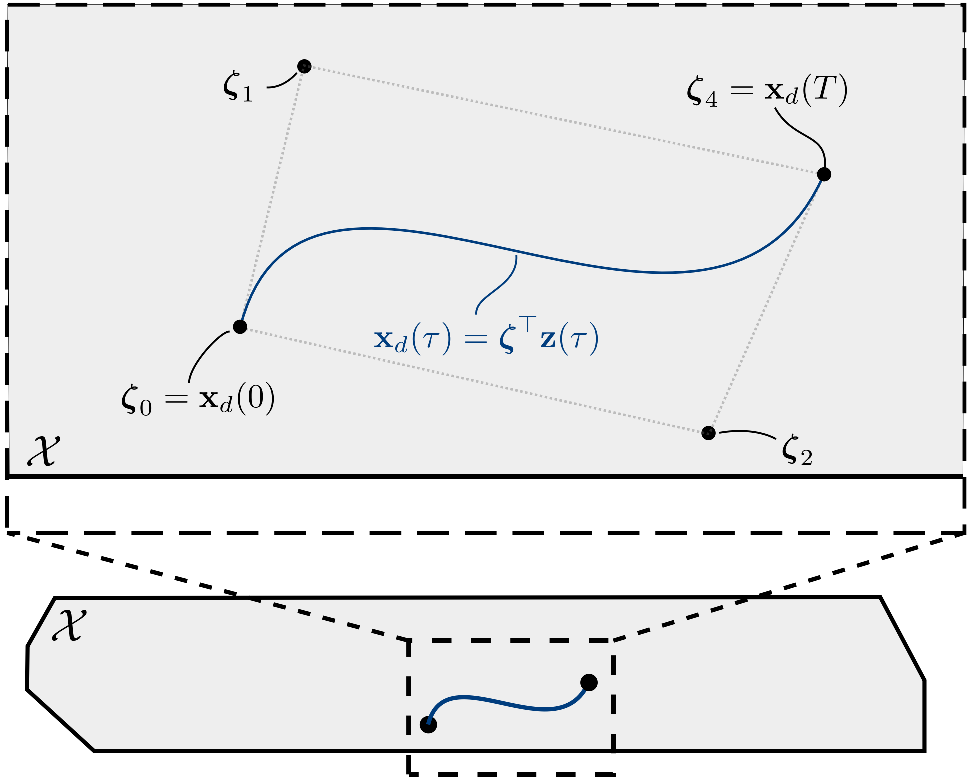
Lemma 2

$\mathbf{x}_d(\tau)$ is dynamically admissible.



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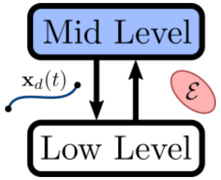

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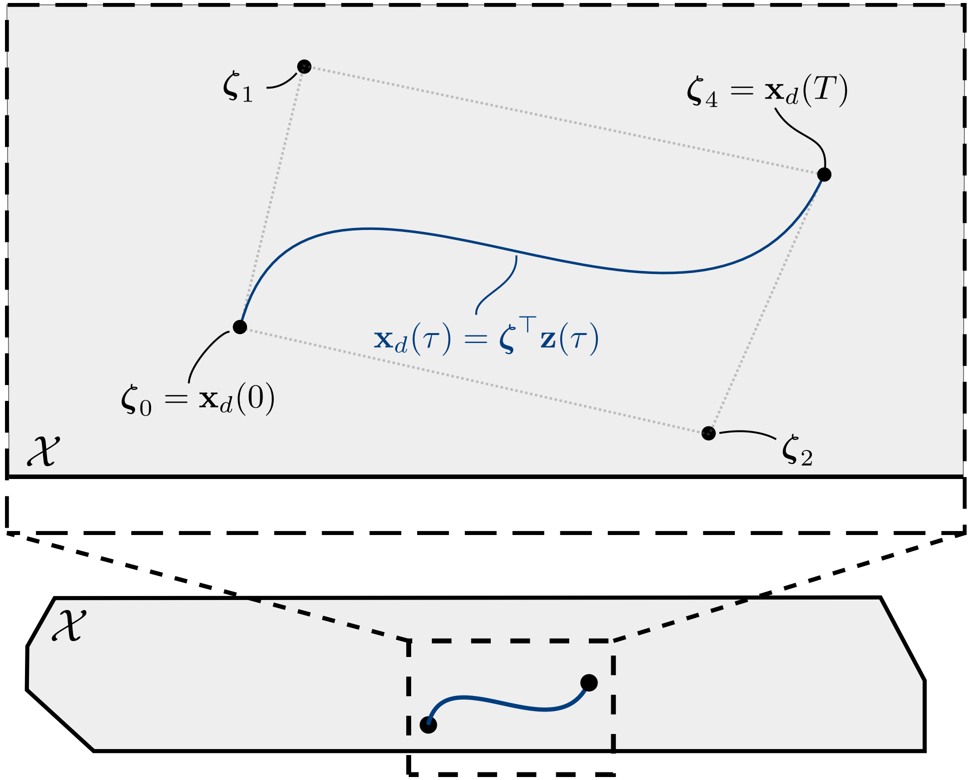
Bézier Curve Properties

- Derivatives of $\mathbf{x}_d(\tau)$ are given by $\mathbf{H}\boldsymbol{\zeta}_i$
- $\mathbf{x}_d(\tau)$ is contained in the convex hull of $\boldsymbol{\zeta}_i$



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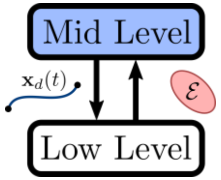

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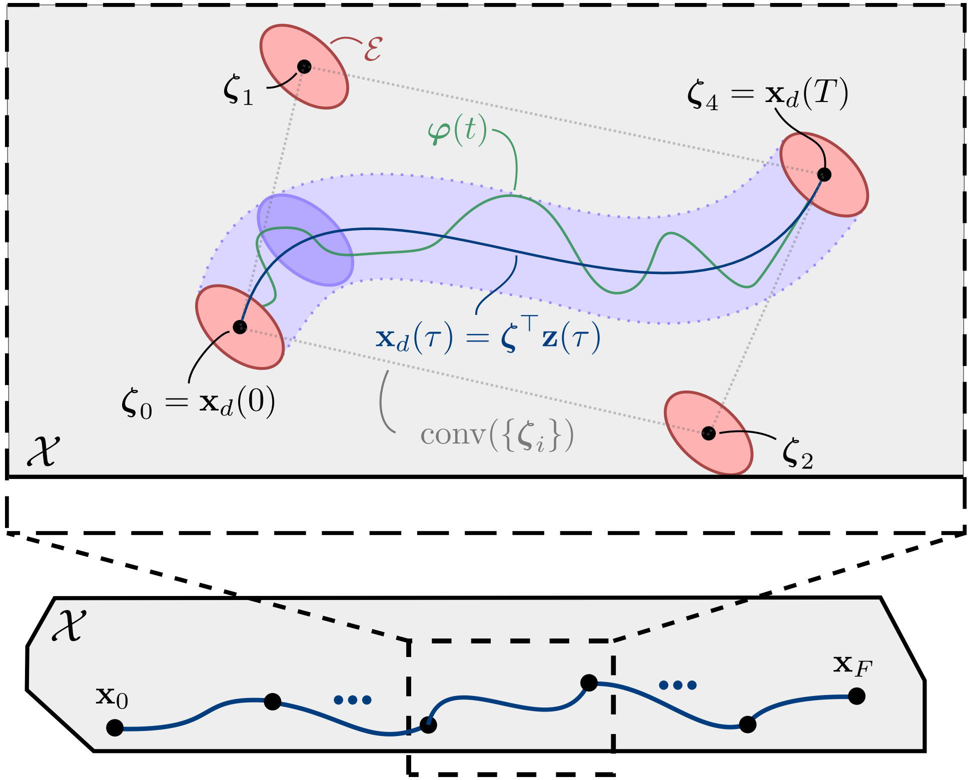

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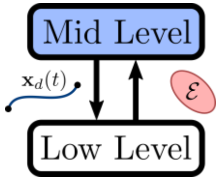

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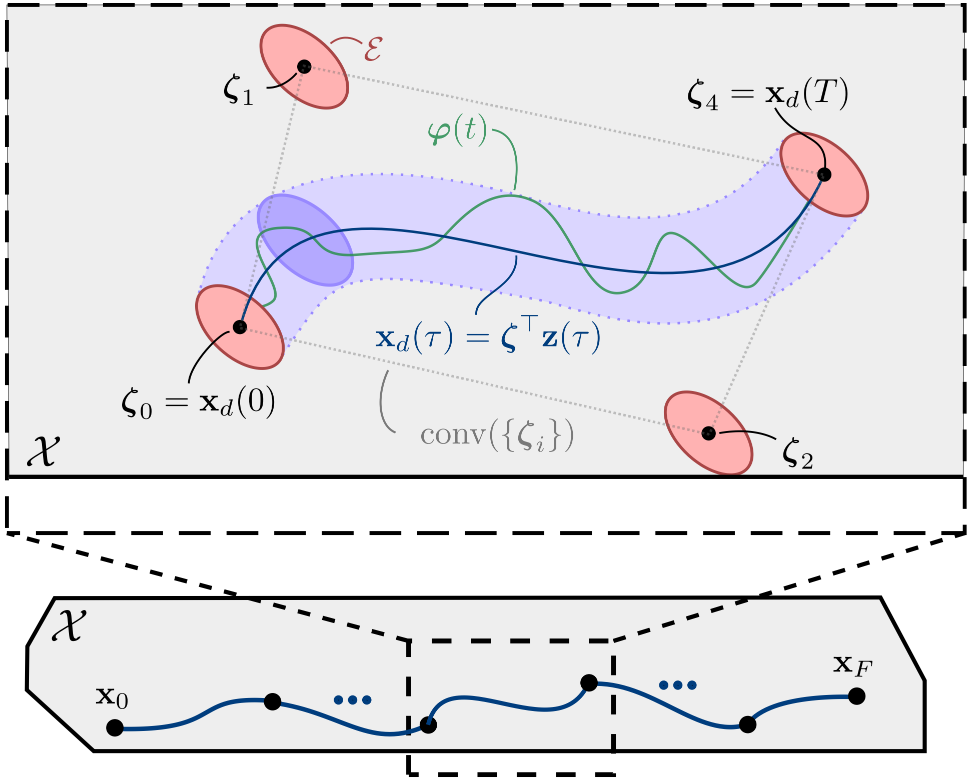
Assumption 2

– $\mathcal{X}$ is a compact, convex polytope



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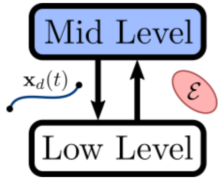

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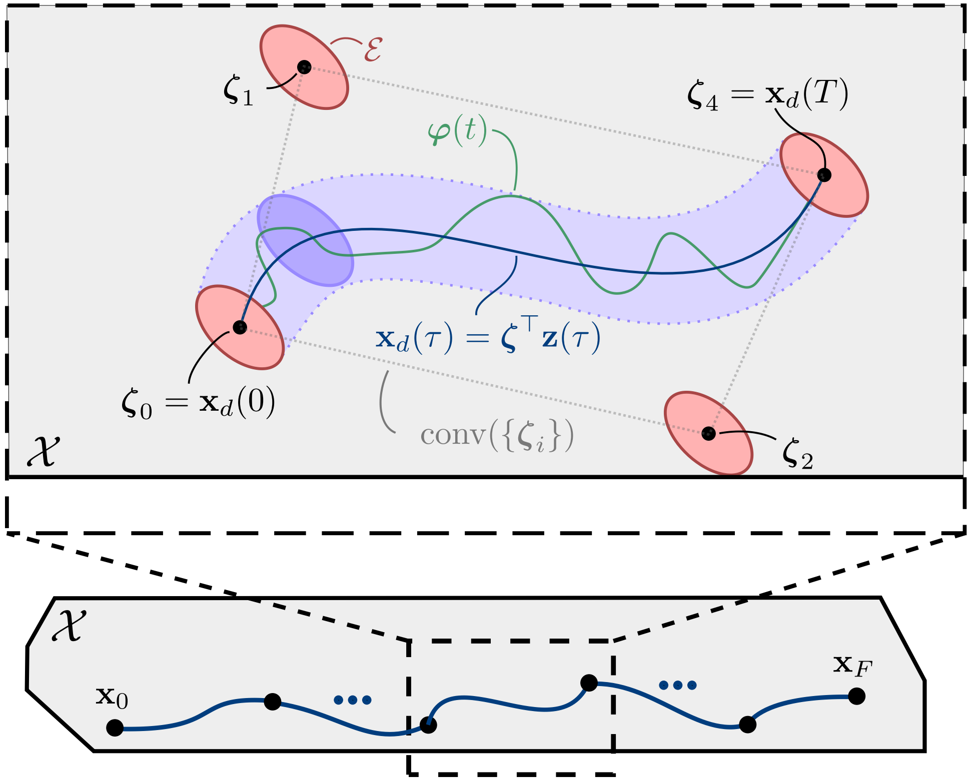
Lemma 3

$$\boldsymbol{\zeta}_i \in \mathcal{X} \ominus \mathcal{E} \implies \varphi(t) \in \mathcal{X}$$



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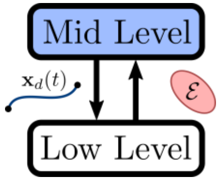

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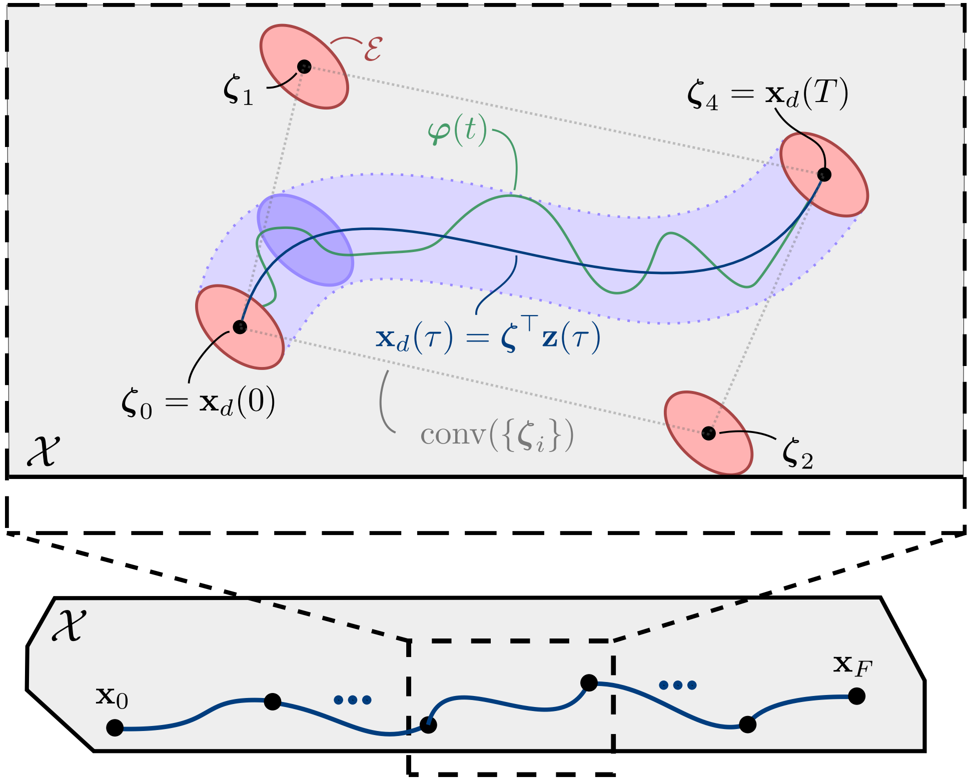
Lemma 4

$$\boldsymbol{\zeta}_i \in \mathcal{X} \ominus \mathcal{E} \implies \varphi(t) \in \mathcal{X}$$

Affine in $\boldsymbol{\zeta}$!



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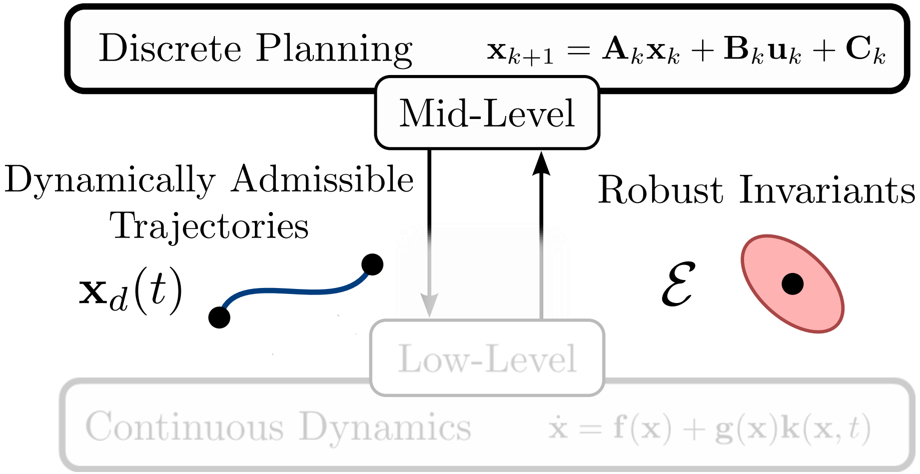

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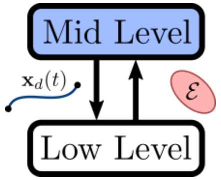
Satisfying Input Constraints



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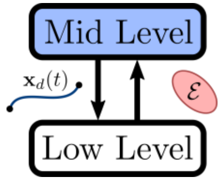

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$$\|k^{\text{clf}}(\mathbf{x}, t)\|_2 \leq \|k^{\text{clf}}(\mathbf{x}, t) - k^{\text{ff}}(\mathbf{x}, t)\|_2 + \|k^{\text{ff}}(\mathbf{x}, t)\|_2 \qquad +0,\ \Delta\text{-inequality}$$

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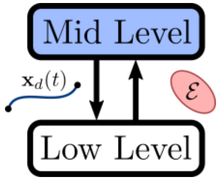

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&\leq \|k^{\text{fbl}}(\mathbf{x},t) - k^{\text{ff}}(\mathbf{x},t)\|_2 + \|k^{\text{ff}}(\mathbf{x},t)\|_2 && \text{Optimality of CLF-QP}
\end{aligned}$$

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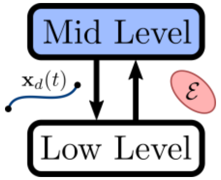

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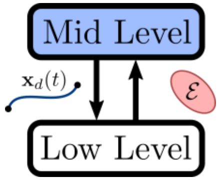

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&\leq \|g^{\dagger}(\mathbf{x})\|_2(\|\mathbf{K}\|_2\|\mathbf{e}\|_2 + \|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2) && \|\cdot\|_2 \text{ Inequalities}
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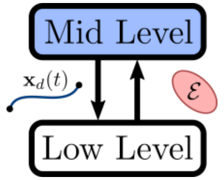

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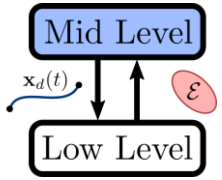
&\leq \|k^{\mathrm{fbl}}(\mathbf{x}, t) - k^{\mathrm{ff}}(\mathbf{x}, t)\|_2 + \|k^{\mathrm{ff}}(\mathbf{x}, t)\|_2 && \text{Optimality of CLF-QP} \\
&= \|g^{\dagger}(\mathbf{x})\mathbf{K}^{\top}\mathbf{e}\|_2 + \|g^{\dagger}(\mathbf{x})(f(\mathbf{x}) - \dot{x}_d^n(t))\|_2 \\
&\leq \|g^{\dagger}(\mathbf{x})\|_2(\|\mathbf{K}\|_2\|\mathbf{e}\|_2 + \|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2) && \|\cdot\|_2 \text{ Inequalities}
\end{aligned}
$$

Satisfying Input Constraints



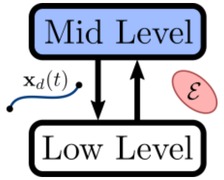
Goal: $\|k^{\text{clf}}(\boldsymbol{\varphi}(t), t)\|_2 \leq u_{\max}$

Approach: $\|k^{\text{clf}}(\boldsymbol{\varphi}(t), t)\|_2 \leq h(\boldsymbol{\zeta}) \leq u_{\max} \qquad (h \text{ convex})$

$$\|k^{\text{clf}}(\mathbf{x}, t)\|_2 \leq \|g^{\dagger}(\mathbf{x})\|_2 (\|\mathbf{K}\|_2 \|\mathbf{e}\|_2 + \|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2)$$

Let $\bar{\mathbf{x}}_k, \mathbf{x}_d(t) \in \mathcal{X}$

Satisfying Input Constraints



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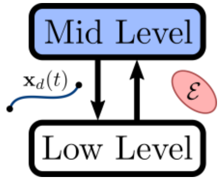
- $g^{\dagger}(\mathbf{x})$

Satisfying Input Constraints



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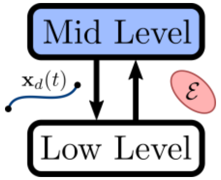
Let $\bar{\mathbf{x}}_k, \mathbf{x}_d(t) \in \mathcal{X}$

$$\|g^\dagger(\mathbf{x})\|_2 \leq \|g^\dagger(\mathbf{x}) - g^\dagger(\mathbf{x}_d(t))\|_2 + \|g^\dagger(\mathbf{x}_d(t)) - g^\dagger(\bar{\mathbf{x}}_k)\|_2 + \|g^\dagger(\bar{\mathbf{x}}_k)\|_2$$

$+0$, Δ-inequalities

- $g^\dagger(\mathbf{x})$

Satisfying Input Constraints



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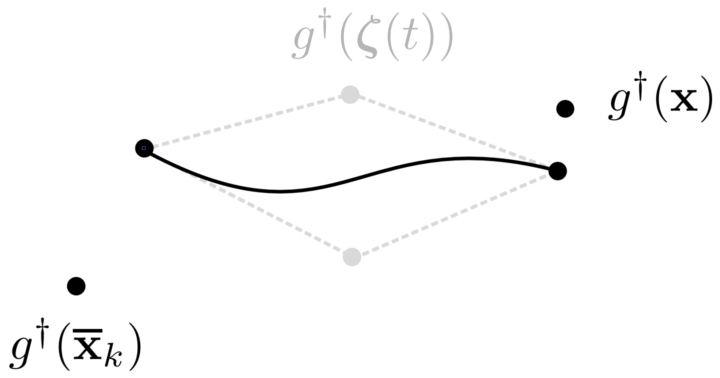
+0, Δ-inequalities

$$\leq L_{g^\dagger}(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|g^\dagger(\bar{\mathbf{x}}_k)\|_2$$

Lipschitz



Noel Csomay-Shanklin
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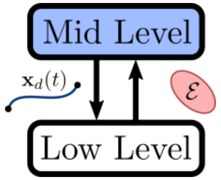
Satisfying Input Constraints



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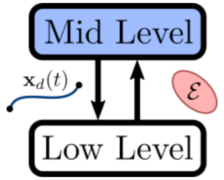
Lipschitz

Satisfying Input Constraints



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$+0$, Δ-inequalities

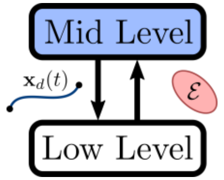
$$\leq L_{g^\dagger}(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|g^\dagger(\bar{\mathbf{x}}_k)\|_2$$

Lipschitz

$$\|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2 \leq L_f(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|f(\bar{\mathbf{x}}_k) - \dot{x}_n^d\|_2$$

—— " ——

Satisfying Input Constraints



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+0, Δ-inequalities

$$\leq L_{g^{\dagger}}(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|g^{\dagger}(\bar{\mathbf{x}}_k)\|_2$$

Lipschitz

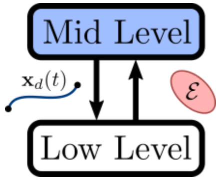
$$\|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2 \leq L_f(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|f(\bar{\mathbf{x}}_k) - \dot{x}_n^d\|_2$$

——— " ———

$$\|\mathbf{e}\|_2 \leq \bar{e}$$

Low Level Controller

Satisfying Input Constraints



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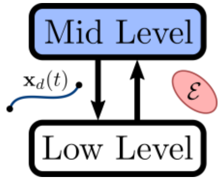
$$\|g^{\dagger}(\mathbf{x})\|_2 \leq L_{g^{\dagger}}(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|g^{\dagger}(\bar{\mathbf{x}}_k)\|_2$$

$$\|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2 \leq L_f(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|f(\bar{\mathbf{x}}_k) - \dot{x}_n^d\|_2$$

$$\|\mathbf{e}\|_2 \leq \bar{e}$$

Satisfying Input Constraints



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$$\|g^{\dagger}(\mathbf{x})\|_2 \leq L_{g^{\dagger}}(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|g^{\dagger}(\bar{\mathbf{x}}_k)\|_2$$

$$\|f(\mathbf{x}) - \dot{x}_d^n(t)\|_2 \leq L_f(\|\mathbf{e}\|_2 + \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2) + \|f(\bar{\mathbf{x}}_k) - \dot{x}_n^d\|_2$$

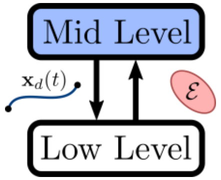
$$\|\mathbf{e}\|_2 \leq \bar{e}$$

$$\|k^{\mathrm{clf}}(\mathbf{x},t)\|_2 \leq \tfrac{1}{2}\boldsymbol{\sigma}(t)^{\top}\mathbf{M}\boldsymbol{\sigma}(t) + \mathbf{N}\boldsymbol{\sigma}(t) + \boldsymbol{\Gamma}$$

$$\boldsymbol{\sigma}(t) = \begin{bmatrix} \|\mathbf{x}_d(t) - \overline{\mathbf{x}}_k\|_2 \\ \|\dot{x}_d^n(t) - f(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \qquad \mathbf{N} = \begin{bmatrix} 2L_{g^{\dagger}}L_f\bar{e} + L_f\|g^{\dagger}(\overline{\mathbf{x}}_k)\|_2 + L_{g^{\dagger}}\|\mathbf{K}\|_2\bar{e} \\ \|g^{\dagger}(\overline{\mathbf{x}}_k)\|_2 + L_{g^{\dagger}}\bar{e} \end{bmatrix}$$

$$\boldsymbol{\Gamma} = \bar{e}(L_{g^{\dagger}}\bar{e} + \|g^{\dagger}(\overline{\mathbf{x}}_k)\|_2)(L_f + \|\mathbf{K}\|_2) \qquad \mathbf{M} = \begin{bmatrix} 2L_{g^{\dagger}}L_f & L_{g^{\dagger}} \\ L_{g^{\dagger}} & 0 \end{bmatrix}$$

Satisfying Input Constraints



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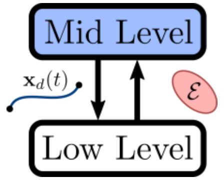
System and Controller Properties
$$\begin{cases} \mathbf{M} = \mathbf{M}(L_{g^\dagger}, L_f) \\ \mathbf{N} = \mathbf{N}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \\ \boldsymbol{\Gamma} = \boldsymbol{\Gamma}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \end{cases}$$

"Trust Region": $\boldsymbol{\sigma}(t) = \begin{bmatrix} \|\mathbf{x}_d(t) - \overline{\mathbf{x}}_k\|_2 \\ \|\dot{\mathbf{x}}_d(t) - \mathbf{f}(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix}$

Satisfying Input Constraints



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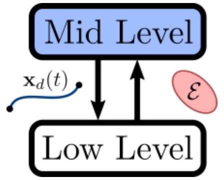

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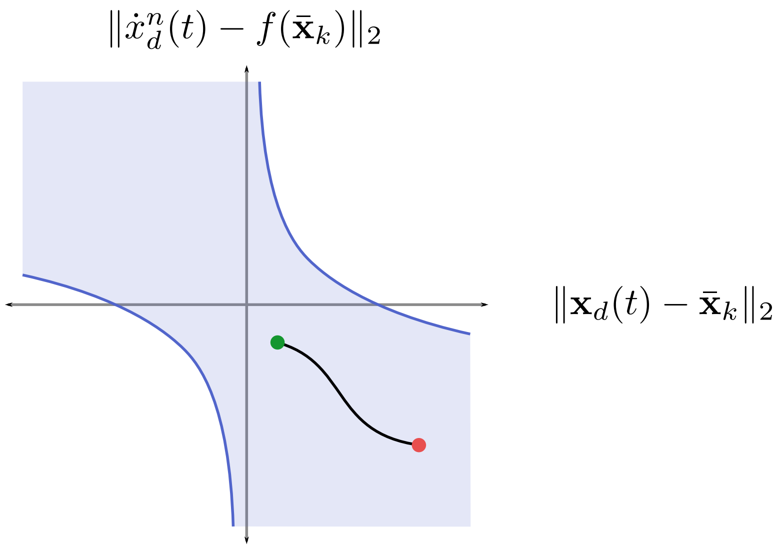

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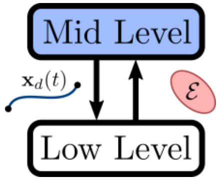

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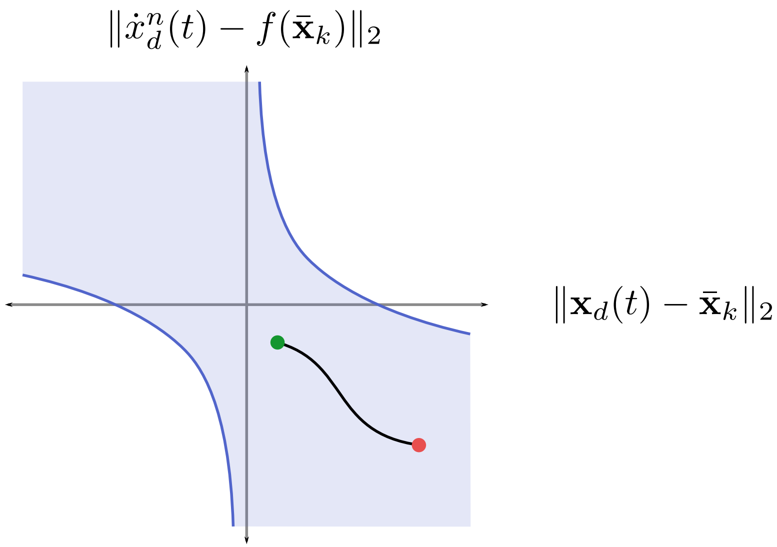
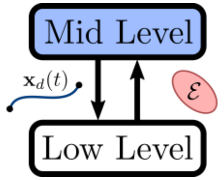


Issues

- Not convex (due to $\mathbf{M}$)
- Requires knowledge of Lipschitz constants
- Needs to be enforced continuously in time



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$$\begin{cases} \mathbf{M} = \pi_{\text{PSD}}\left(\mathbf{M}(L_{g^\dagger}, L_f)\right) \succeq \mathbf{M}(L_{g^\dagger}, L_f) \\ \mathbf{N} = \mathbf{N}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \\ \boldsymbol{\Gamma} = \boldsymbol{\Gamma}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \end{cases}$$

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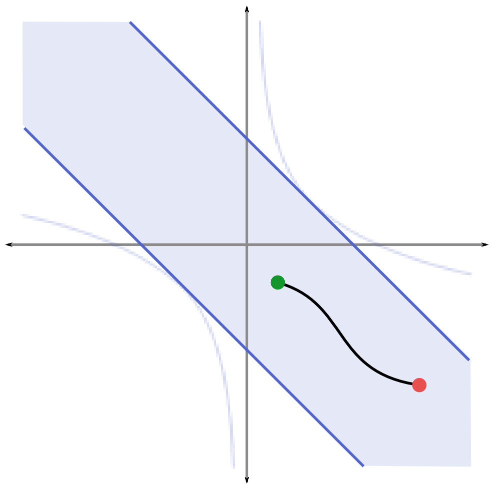

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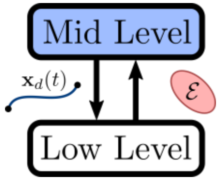

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Tuning Knobs
$$\begin{cases} \mathbf{M} = \pi_{\text{PSD}}\left(\mathbf{M}(\alpha,\beta)\right) \succeq \mathbf{M}(L_{g^\dagger}, L_f) \\ \mathbf{N} = \mathbf{N}(\alpha,\beta,\mathcal{E},\mathbf{K}) \succeq \mathbf{N}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \\ \boldsymbol{\Gamma} = \Gamma(\alpha,\beta,\mathcal{E},\mathbf{K}) \geq \boldsymbol{\Gamma}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \end{cases}$$

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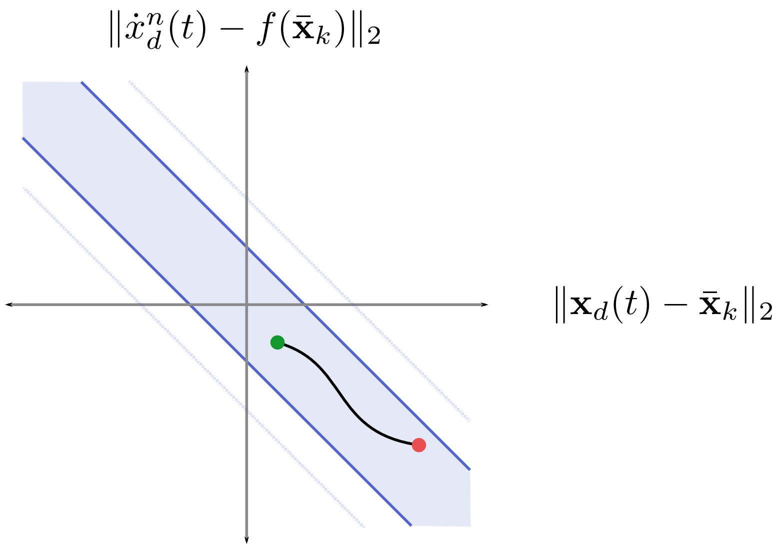

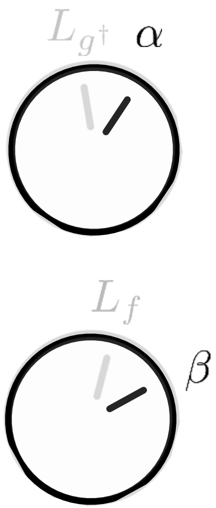

Theorem 1

For α, β sufficiently large, $\mathbf{M}$, $\mathbf{N}$, and $\boldsymbol{\Gamma}$ respect the above ordering

Issues

- Not convex (due to $\mathbf{M}$) ✓
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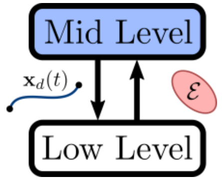
Satisfying Input Constraints



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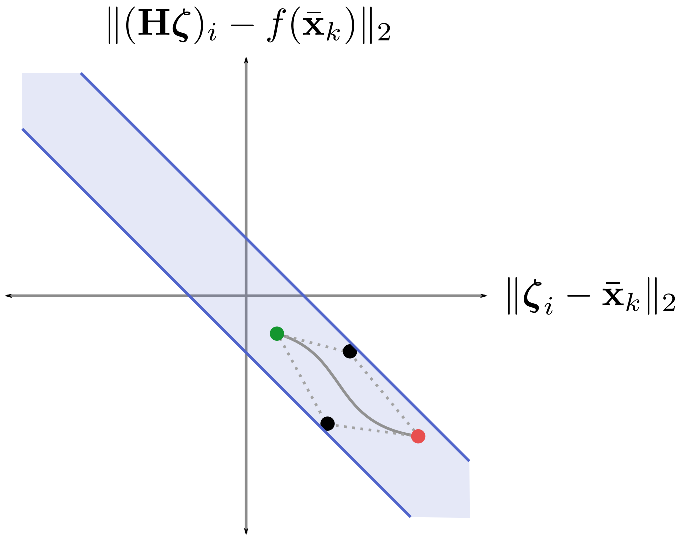
Approach: $\|k^{\text{clf}}(\varphi(t),t)\|_2 \leq h(\zeta) \leq u_{\max}$ $\quad$ (h convex)

$$\|k^{\text{clf}}(\mathbf{x},t)\|_2 \leq \tfrac{1}{2}\boldsymbol{\sigma}(t)^\top \mathbf{M}\boldsymbol{\sigma}(t) + \mathbf{N}\boldsymbol{\sigma}(t) + \boldsymbol{\Gamma} \leq u_{\max}$$

$$\text{Tuning Knobs} \begin{cases} \mathbf{M} = \pi_{\text{PSD}}\left(\mathbf{M}(\alpha,\beta)\right) \succeq \mathbf{M}(L_{g^\dagger}, L_f) \\ \mathbf{N} = \mathbf{N}(\alpha,\beta,\mathcal{E},\mathbf{K}) \succeq \mathbf{N}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \\ \boldsymbol{\Gamma} = \Gamma(\alpha,\beta,\mathcal{E},\mathbf{K}) \geq \boldsymbol{\Gamma}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \end{cases}$$

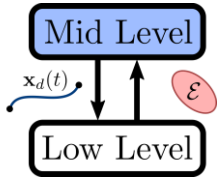
$$\boldsymbol{\sigma}(t) = \begin{bmatrix} \|\mathbf{x}_d(t) - \bar{\mathbf{x}}_k\|_2 \\ \|\dot{\mathbf{x}}_d(t) - \mathbf{f}(\bar{\mathbf{x}}_k)\|_2 \end{bmatrix} \leq \max_i \begin{bmatrix} \|(\boldsymbol{\zeta})_i - \bar{\mathbf{x}}_k\|_2, \\ \|(\mathbf{H}\boldsymbol{\zeta})_i - f(\bar{\mathbf{x}}_k)\|_2 \end{bmatrix}$$



Issues

- Not convex (due to $\mathbf{M}$) ✓
- Requires knowledge of Lipschitz constants ✓
- Needs to be enforced continuously in time

Satisfying Input Constraints



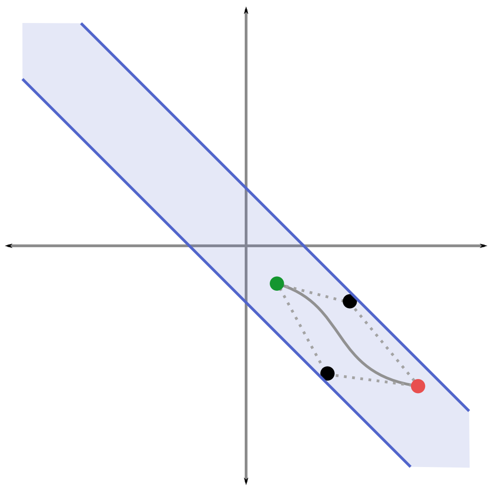
Goal: $\|k^{\text{clf}}(\boldsymbol{\varphi}(t),t)\|_2 \leq u_{\max}$

Approach: $\|k^{\text{clf}}(\boldsymbol{\varphi}(t),t)\|_2 \leq h(\boldsymbol{\zeta}) \leq u_{\max}$ (h convex)

$$\|k^{\text{clf}}(\mathbf{x},t)\|_2 \leq \tfrac{1}{2}\boldsymbol{\sigma}(t)^\top \mathbf{M}\boldsymbol{\sigma}(t) + \mathbf{N}\boldsymbol{\sigma}(t) + \boldsymbol{\Gamma} \leq u_{\max}$$

$$\text{Tuning Knobs} \begin{cases} \mathbf{M} = \pi_{\text{PSD}}\left(\mathbf{M}(\alpha,\beta)\right) \succeq \mathbf{M}(L_{g^\dagger}, L_f) \\ \mathbf{N} = \mathbf{N}(\alpha,\beta,\mathcal{E},\mathbf{K}) \succeq \mathbf{N}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \\ \boldsymbol{\Gamma} = \Gamma(\alpha,\beta,\mathcal{E},\mathbf{K}) \geq \boldsymbol{\Gamma}(L_{g^\dagger}, L_f, \mathcal{E}, \mathbf{K}) \end{cases}$$

$$\boldsymbol{\sigma}(t) = \begin{bmatrix} \|\mathbf{x}_d(t) - \overline{\mathbf{x}}_k\|_2 \\ \|\dot{\mathbf{x}}_d(t) - \mathbf{f}(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \leq \max_i \begin{bmatrix} \|(\boldsymbol{\zeta})_i - \overline{\mathbf{x}}_k\|_2, \\ \|(\mathbf{H}\boldsymbol{\zeta})_i - f(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix}$$



Lemma 6

$$\begin{bmatrix} \|(\boldsymbol{\zeta})_i - \overline{\mathbf{x}}_k\|_2, \\ \|(\mathbf{H}\boldsymbol{\zeta})_i - f(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \leq \mathbf{s}, \ \forall i \qquad \tfrac{1}{2}\mathbf{s}^\top\mathbf{M}\mathbf{s} + \mathbf{N}\mathbf{s} + \boldsymbol{\Gamma} \leq u_{\max}$$

$$\Downarrow$$

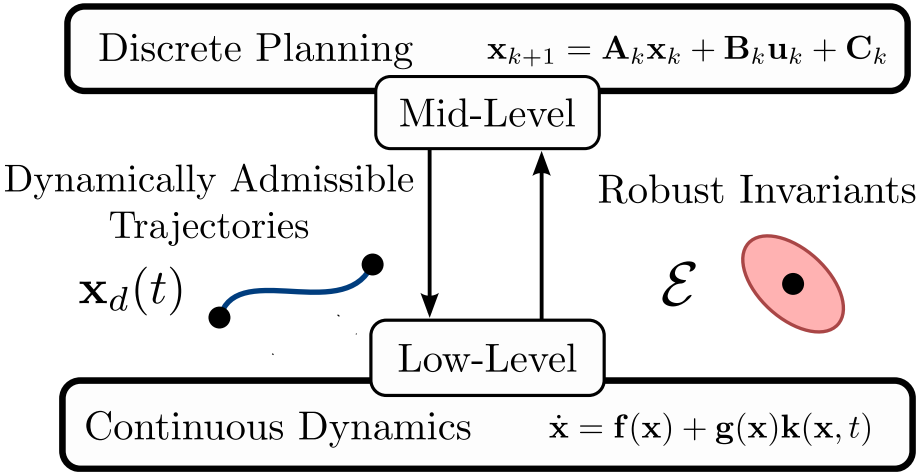
$$\boldsymbol{\sigma}(t) = \begin{bmatrix} \|\mathbf{x}_d(t) - \overline{\mathbf{x}}_k\|_2 \\ \|\dot{\mathbf{x}}_d(t) - \mathbf{f}(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \qquad \tfrac{1}{2}\boldsymbol{\sigma}(t)^\top\mathbf{M}\boldsymbol{\sigma}(t) + \mathbf{N}\boldsymbol{\sigma}(t) + \boldsymbol{\Gamma} \leq u_{\max}$$

Issues

- Not convex (due to $\mathbf{M}$) ✓
- Requires knowledge of Lipschitz constants ✓
- Needs to be enforced continuously in time ✓

Overview

- Designing a Feedback Controller
- Producing Trajectories and Satisfying State Constraints
- Satisfying Input Constraints
- Multi-Rate Architechture



Multi-Rate Architechture

$$
\begin{aligned}
\min_{\substack{u_k,\mathbf{x}_k\\ \mathbf{s}_k,\boldsymbol{\zeta}_k}} \quad & \sum_{k=0}^{N-1} h(\mathbf{x}_k, u_k) + J(\mathbf{x}_N) && \text{(FTOCP)}\\
\text{s.t.} \quad & \mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k u_k + \mathbf{C}_k, && \text{(Dynamics)}\\
& \mathbf{x}_0 \in \mathbf{x}(t) \oplus \mathcal{E}, && \text{(Initial Condition)}\\
& \mathbf{x}_N = \mathbf{0}, && \text{(Terminal Constraint)}\\
& (\boldsymbol{\zeta}_k) = \begin{bmatrix} \mathbf{x}_k^\top & \mathbf{x}_{k+1}^\top \end{bmatrix} \mathbf{D}^{-1}, && \text{(Trajectory Construction)}\\
& (\boldsymbol{\zeta}_k)_i \in \mathcal{X} \ominus \mathcal{E}, \qquad \forall i \in \mathcal{I} && \text{(State Constraint)}\\
& \begin{bmatrix} \|(\boldsymbol{\zeta}_k)_i - \overline{\mathbf{x}}_k\|_2 \\ \|(\mathbf{H}\boldsymbol{\zeta}_k)_i - f(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \le \mathbf{s}_k, \qquad \forall i \in \mathcal{I} && \text{(Input Constraint)}\\
& \frac{1}{2}\mathbf{s}_k^\top \mathbf{M}_{\alpha,\beta}\mathbf{s}_k + \mathbf{N}_{\alpha,\beta}^\top \mathbf{s}_k + \Gamma_{\alpha,\beta} \le u_{\max}, && \text{(Input Constraint)}
\end{aligned}
$$

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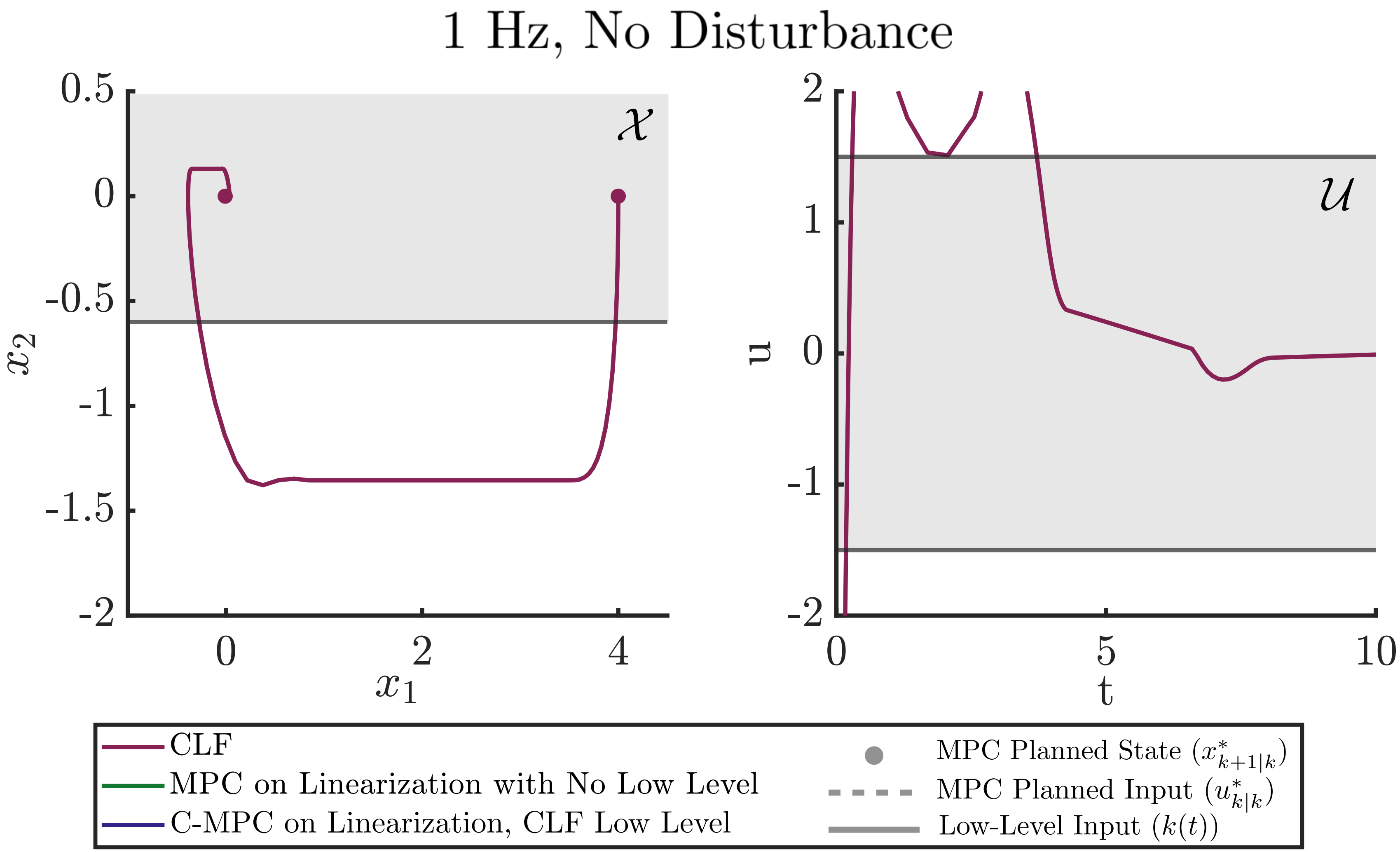
Multi-Rate Architechture

$$
\begin{aligned}
\min_{\substack{u_k,\mathbf{x}_k \\ \mathbf{s}_k,\boldsymbol{\zeta}_k}} \quad & \sum_{k=0}^{N-1} h(\mathbf{x}_k, u_k) + J(\mathbf{x}_N) \\
\text{s.t.} \quad & \mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k u_k + \mathbf{C}_k, \\
& \mathbf{x}_0 \in \mathbf{x}(t) \oplus \mathcal{E}, \\
& \mathbf{x}_N = \mathbf{0}, \\
& (\boldsymbol{\zeta}_k) = \begin{bmatrix} \mathbf{x}_k^\top & \mathbf{x}_{k+1}^\top \end{bmatrix} \mathbf{D}^{-1}, \\
& (\boldsymbol{\zeta}_k)_i \in \mathcal{X} \ominus \mathcal{E}, \qquad \forall i \in \mathcal{I} \\
& \begin{bmatrix} \|(\boldsymbol{\zeta}_k)_i - \overline{\mathbf{x}}_k\|_2 \\ \|(\mathbf{H}\boldsymbol{\zeta}_k)_i - f(\overline{\mathbf{x}}_k)\|_2 \end{bmatrix} \leq \mathbf{s}_k, \qquad \forall i \in \mathcal{I} \\
& \frac{1}{2}\mathbf{s}_k^\top \mathbf{M}_{\alpha,\beta} \mathbf{s}_k + \mathbf{N}_{\alpha,\beta}^\top \mathbf{s}_k + \Gamma_{\alpha,\beta} \leq u_{\max}
\end{aligned}
$$

$\mathbf{x}_d(t)$ ↓ $\quad$ ↑ $\mathcal{E}$

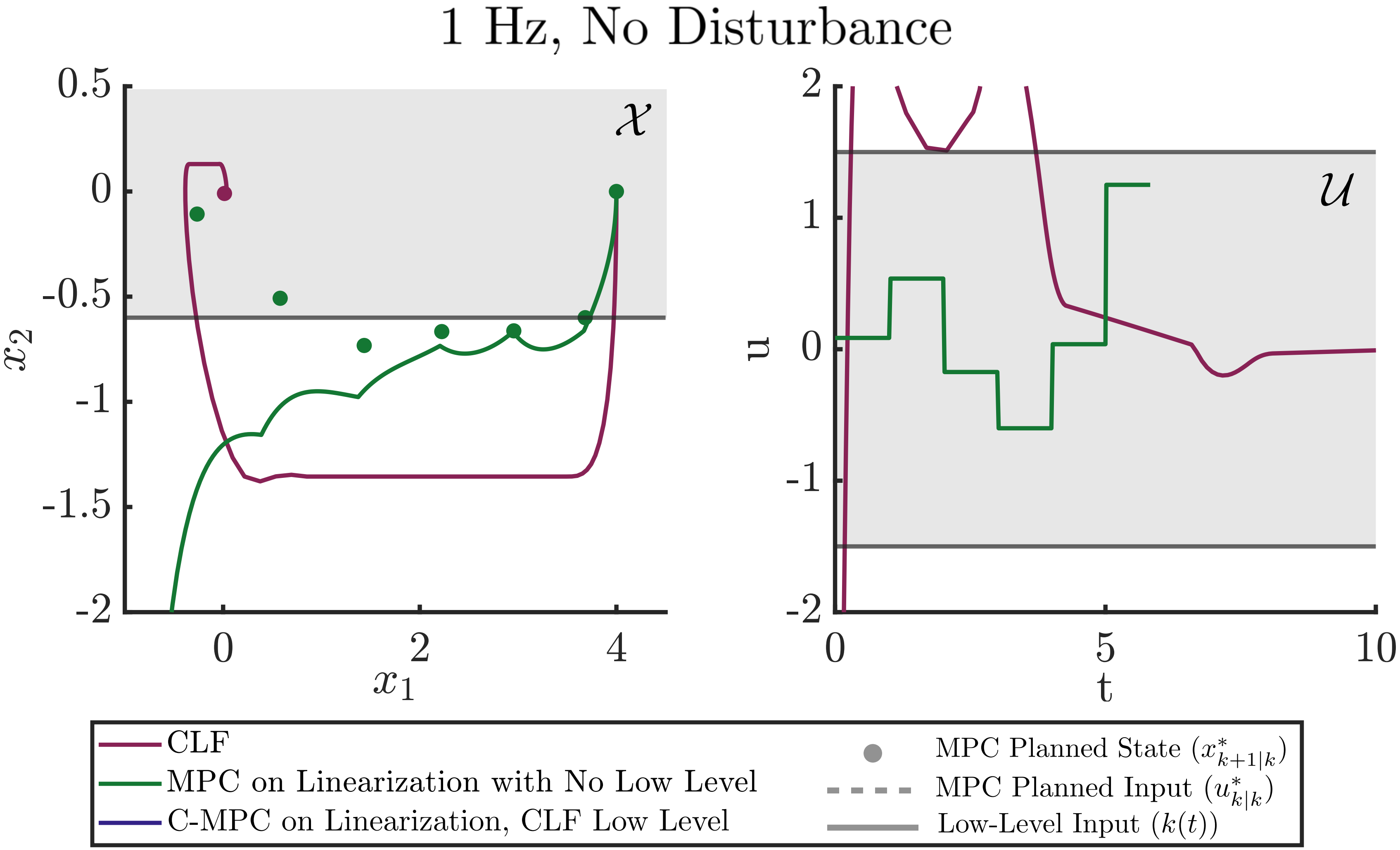
$$
\begin{aligned}
k^{\text{clf}}(\mathbf{x}, t) = \ & \underset{u \in \mathbb{R}}{\operatorname{argmin}} \ \frac{1}{2}\|u - k^{\text{ff}}(\mathbf{x}, t)\|_2^2 \\
& \text{s.t. } \dot{V}(\mathbf{x}, u, t) \leq -\gamma V(\mathbf{x}, t)
\end{aligned}
$$

Results



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \sin(x_1) + x_2^3\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u$$

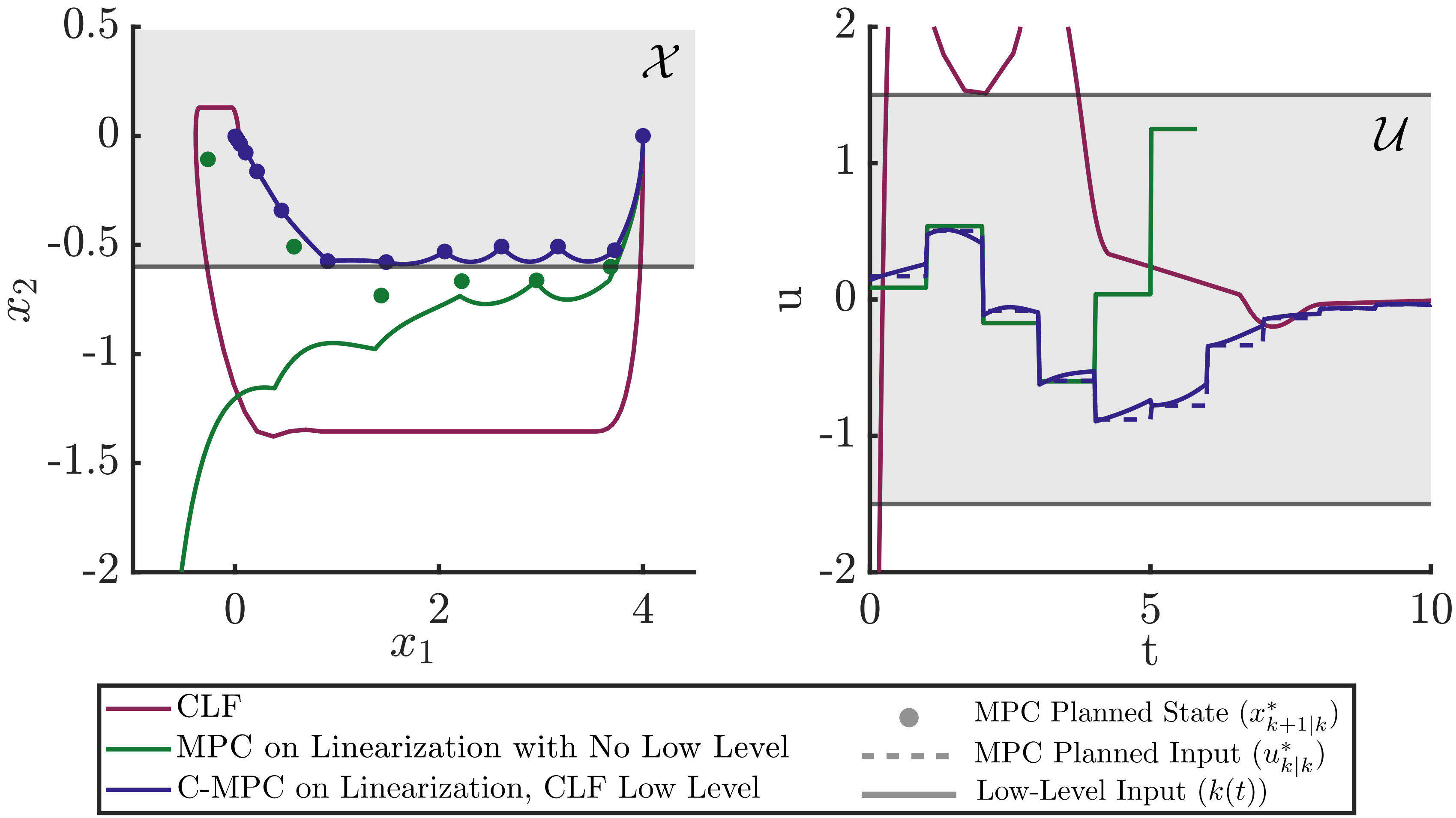
Results



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \sin(x_1) + x_2^3\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

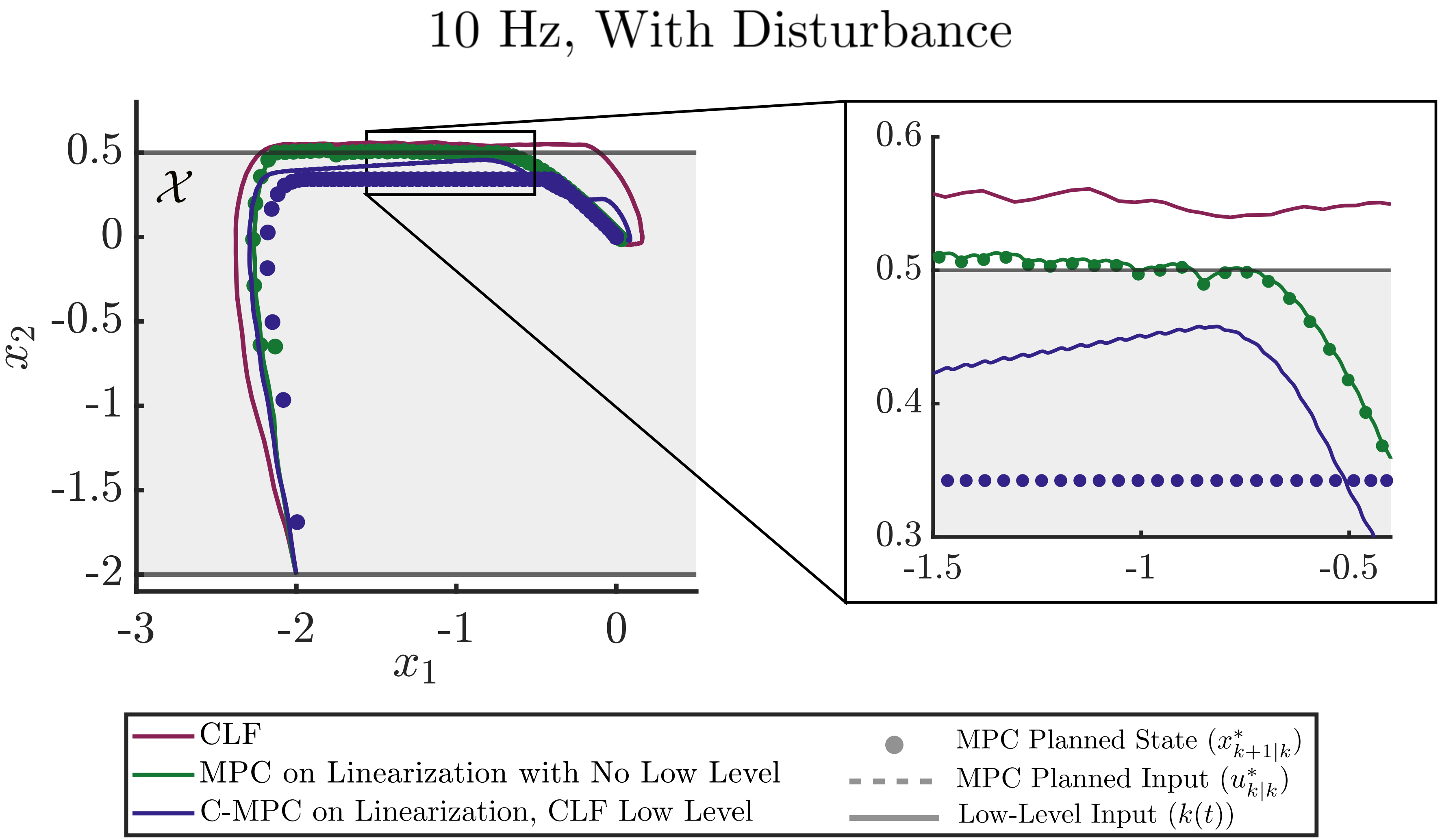
Results

1 Hz, No Disturbance



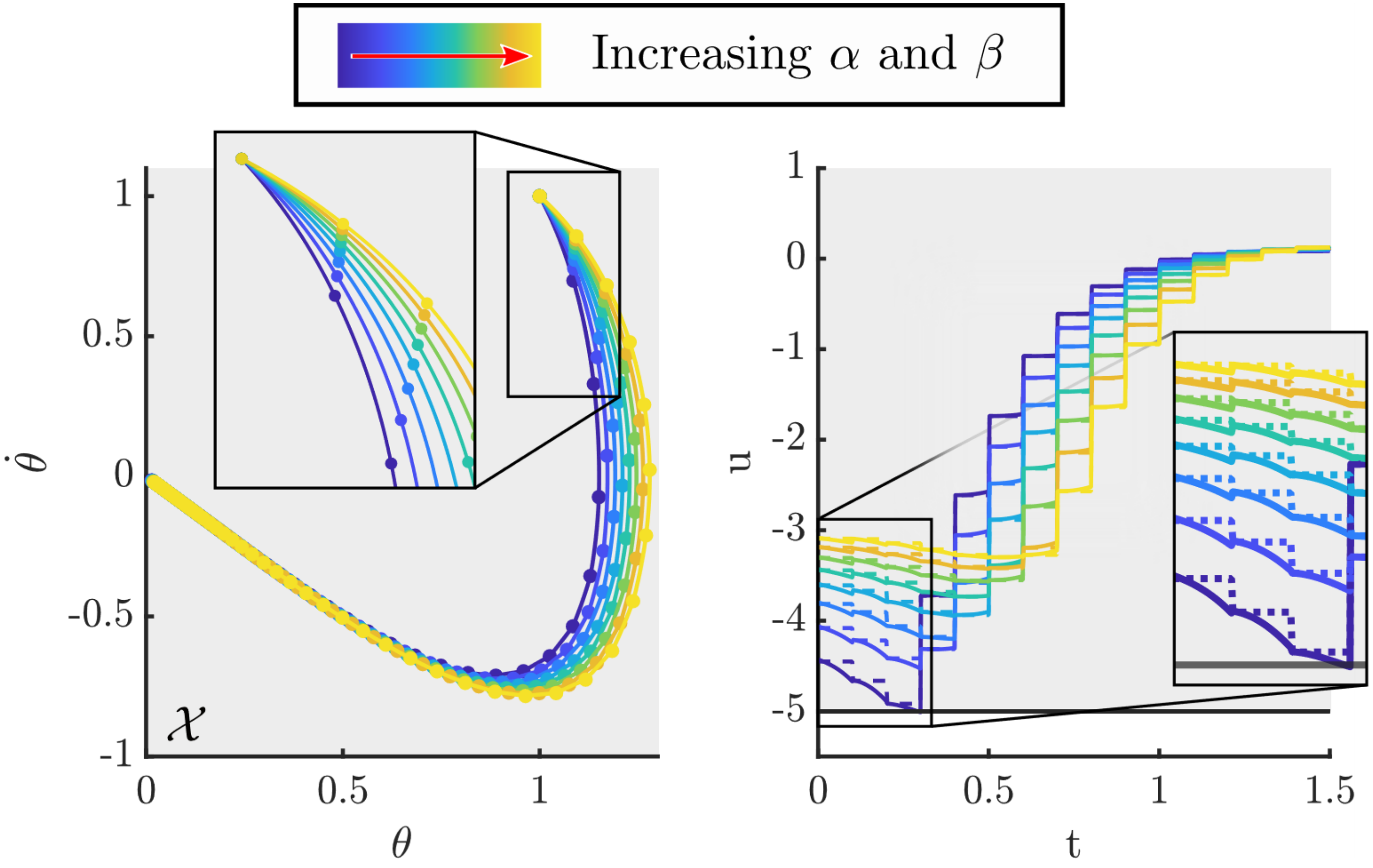
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \sin(x_1) + x_2^3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Results



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \sin(x_1)+x_2^3\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix}$$

Results



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \sin(x_1)+x_2^3\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix}$$

Conclusion

Summary:

– Developed a multi-rate architechture integrating CLFs and MPC for robust state and input constrained nonlinear stabilization

– Used convexity properties of Bézier curves to enable tractable online planning with guarantees

Future Work:

– Extension to Sampled-Data and MIMO settings

– Systems with underactuation

– Adding high level

– Hardware demonstration