Episodic Learning for Safe Bipedal Locomotion with Control Barrier Functions and Projection-to-State Safety

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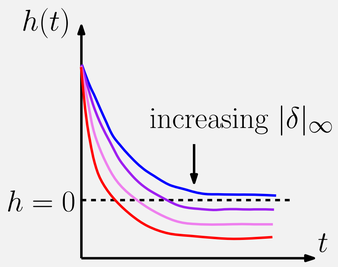
Abstract

- Control Barrier Functions encode safety objectives
- Projection-to-State Safety quantifies constraint degradation with model uncertainty
- Data is collected and parameters are fit episodically
- Safe traversal of stepping stones is demonstrated in simulation and experimentally

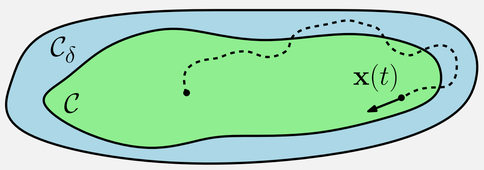
Projection-to-State Safety



$$\dot{x} = \widehat{f}(x) + \widehat{g}(x)u + \underbrace{f(x) - \widehat{f}(x)}_{b(x)} + \underbrace{(g(x) - \widehat{g}(x))}_{A(x)}u$$

$$\dot{h}(x, k(x)) = \underbrace{\nabla h(x)(\widehat{f}(x) + \widehat{g}(x)k(x))}_{\widehat{\dot{h}}(x,k(x))} + \underbrace{\nabla h(x)b(x) + \nabla h(x)A(x)k(x)}_{\delta(x)}$$

$$\delta(x) = \underbrace{\dot{h}(x, k(x))}_{\text{Measured}} - \underbrace{\widehat{\dot{h}}(x, k(x))}_{\text{Estimated}}$$



$$C \triangleq \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

$$C_\delta \triangleq \{x \in \mathbb{R}^n : h(x) + \gamma(|\delta|_\infty) \geq 0\}$$

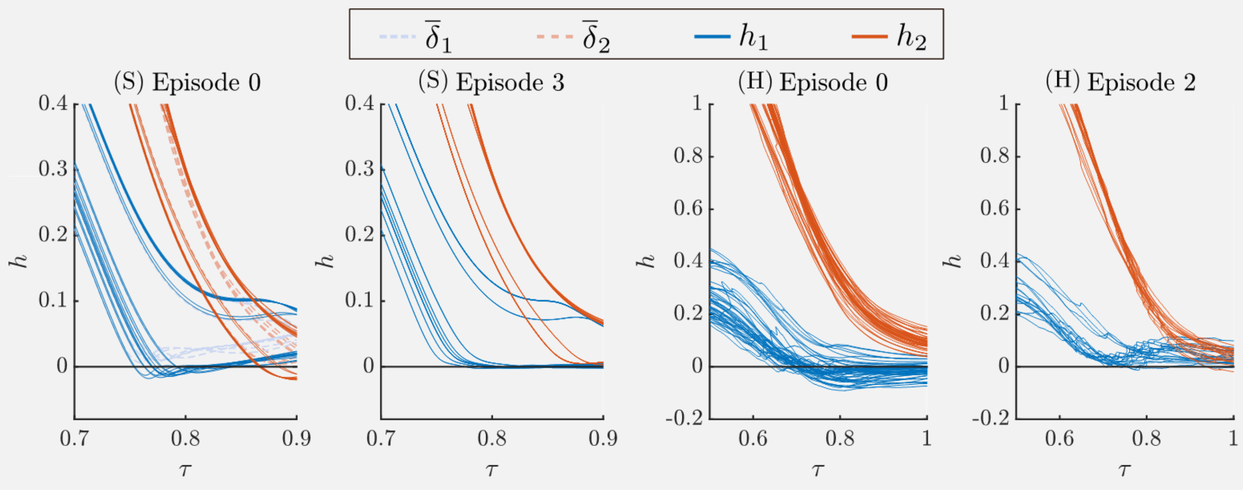
Final Controller (SS-QP)

$$k(x) = \underset{u \in \mathbb{R}^m}{\text{argmin}} \quad \frac{1}{2}\|u - k_{PD}(x)\|_2^2$$

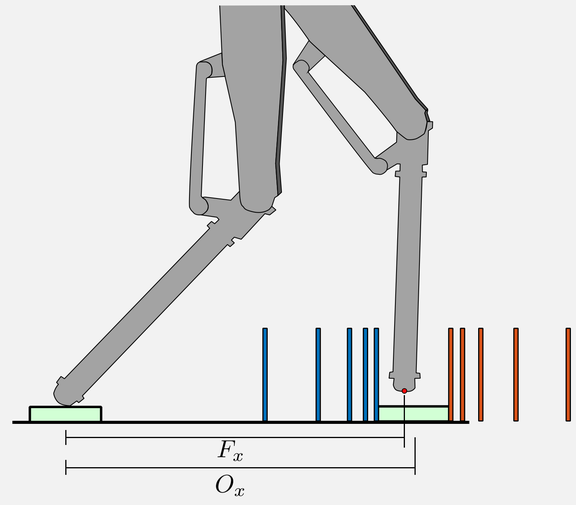
$$\text{s.t.} \quad L_{\hat{f}}^2 h_1(x) + L_{\hat{g}} L_{\hat{f}} h_1(x) u + \alpha_e L_{\hat{f}} h_1(x) + \widehat{\delta}_1(x) \geq -\alpha(h_{e,1}(x))$$

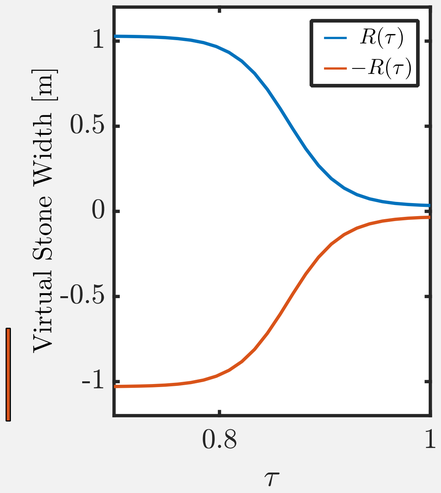
$$L_{\hat{f}}^2 h_2(x) + L_{\hat{g}} L_{\hat{f}} h_2(x) u + \alpha_e L_{\hat{f}} h_2(x) + \widehat{\delta}_2(x) \geq -\alpha(h_{e,2}(x))$$

Results



Control Barrier Functions

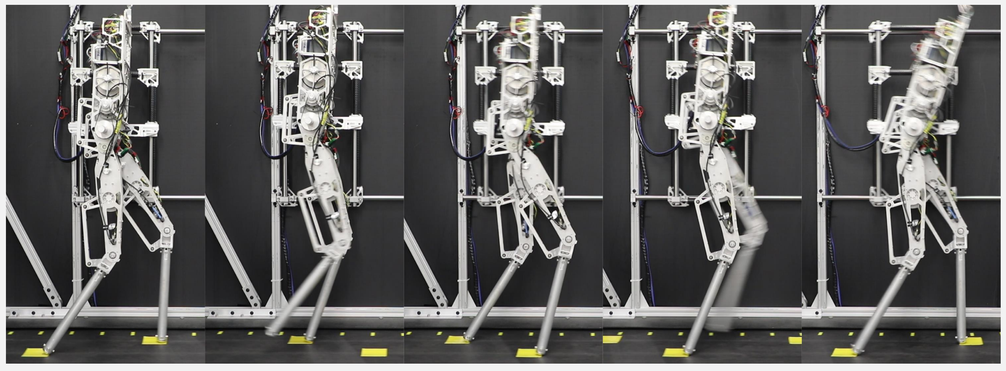




$$h_1(q) = R(\tau(q)) - (O_x - F_x(q)) \qquad h_2(q) = R(\tau(q)) + (O_x - F_x(q))$$

Episodes

Data Collection



Parameter Fitting (ERM)

$$\widehat{\delta}(x) = \underset{\widehat{\delta} \in \mathcal{H}}{\text{argmin}} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}\left(\widehat{\dot{h}}(x_i, k(x_i)) + \widehat{\delta}(x_i), \dot{h}_i\right)$$

$\widehat{\delta}$-CBF-QP

$$k(x) = \underset{u \in \mathbb{R}^m}{\text{argmin}} \quad \frac{1}{2}\|u - k_d(x)\|_2^2$$

$$\text{s.t.} \quad \widehat{\dot{h}}(x, u) + \widehat{\delta}(x) \geq -\alpha(h(x))$$

for $j = 1, \ldots, T$ **do**
  $\mathfrak{D}_j \leftarrow$ `experiment` $(\mathbf{x}_0, \mathbf{k}_{j-1})$
  $\widehat{\delta} \leftarrow$ `ERM` $(\mathcal{H}, \mathcal{L}, \mathfrak{D}_j, \widehat{\dot{h}}_0)$
  $\widehat{\dot{h}}_j \leftarrow \widehat{\dot{h}}_0 + \widehat{\delta}$
  $\mathbf{k}_j \leftarrow \widehat{\delta}$-`CBF-QP` $(\widehat{\dot{h}}_j)$
end

https://youtu.be/BvpVY2JRdVY